



# Particle based simulation of steel components

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# Aim

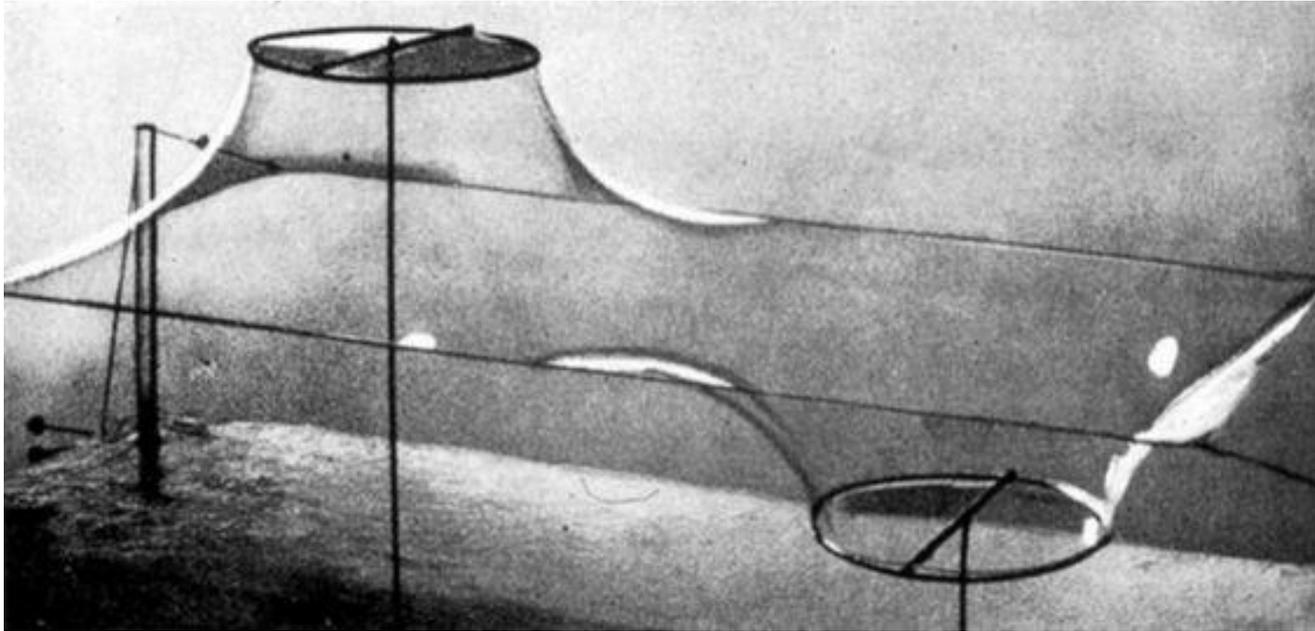
To contribute with development of methods that can be used to support design of high quality and material efficient structural components.

Motivated by the challenge to make building that last and to reduce embodied carbon in the structures that we build.





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Soap bubble experiments by Frei Otto



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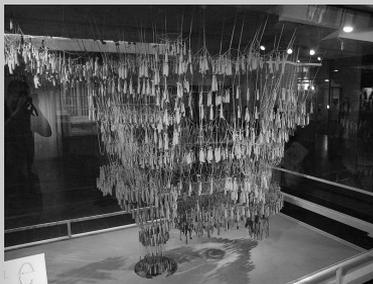
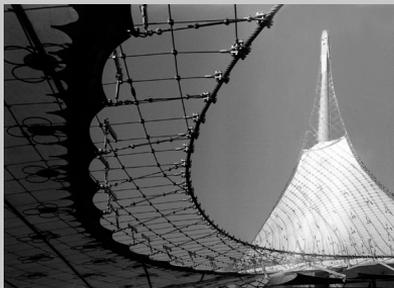
# Introduction



Render: Foster + Partners



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The British Museum Great Court Roof, photo:  
C. R. K. Lander



Figure 2. Level change function.

Figure 3. Function with finite curvature at corners

$$\frac{\left(1 - \frac{x}{b}\right)\left(1 + \frac{y}{b}\right)\left(1 - \frac{y}{c}\right)\left(1 + \frac{x}{d}\right)}{\left(1 - \frac{ax}{rb}\right)\left(1 + \frac{ay}{rb}\right)\left(1 - \frac{ay}{rc}\right)\left(1 + \frac{ax}{rd}\right)}$$

$$\left(\frac{r}{a} - 1\right)\left(1 - \frac{x}{b}\right)\left(1 + \frac{x}{b}\right)\left(1 - \frac{y}{c}\right)\left(1 + \frac{y}{d}\right)$$



Figure 4. Function with conical corners

$$\frac{1 - \frac{x}{r}}{\frac{\sqrt{(b-x)^2 + (c-y)^2}}{(b-x)(c-y)} + \frac{\sqrt{(b-x)^2 + (d+y)^2}}{(b-x)(d+y)} + \frac{\sqrt{(b+x)^2 + (c-y)^2}}{(b+x)(c-y)} + \frac{\sqrt{(b+x)^2 + (d+y)^2}}{(b+x)(d+y)}}$$

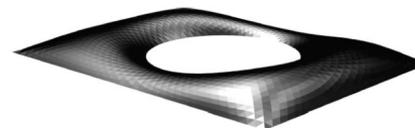
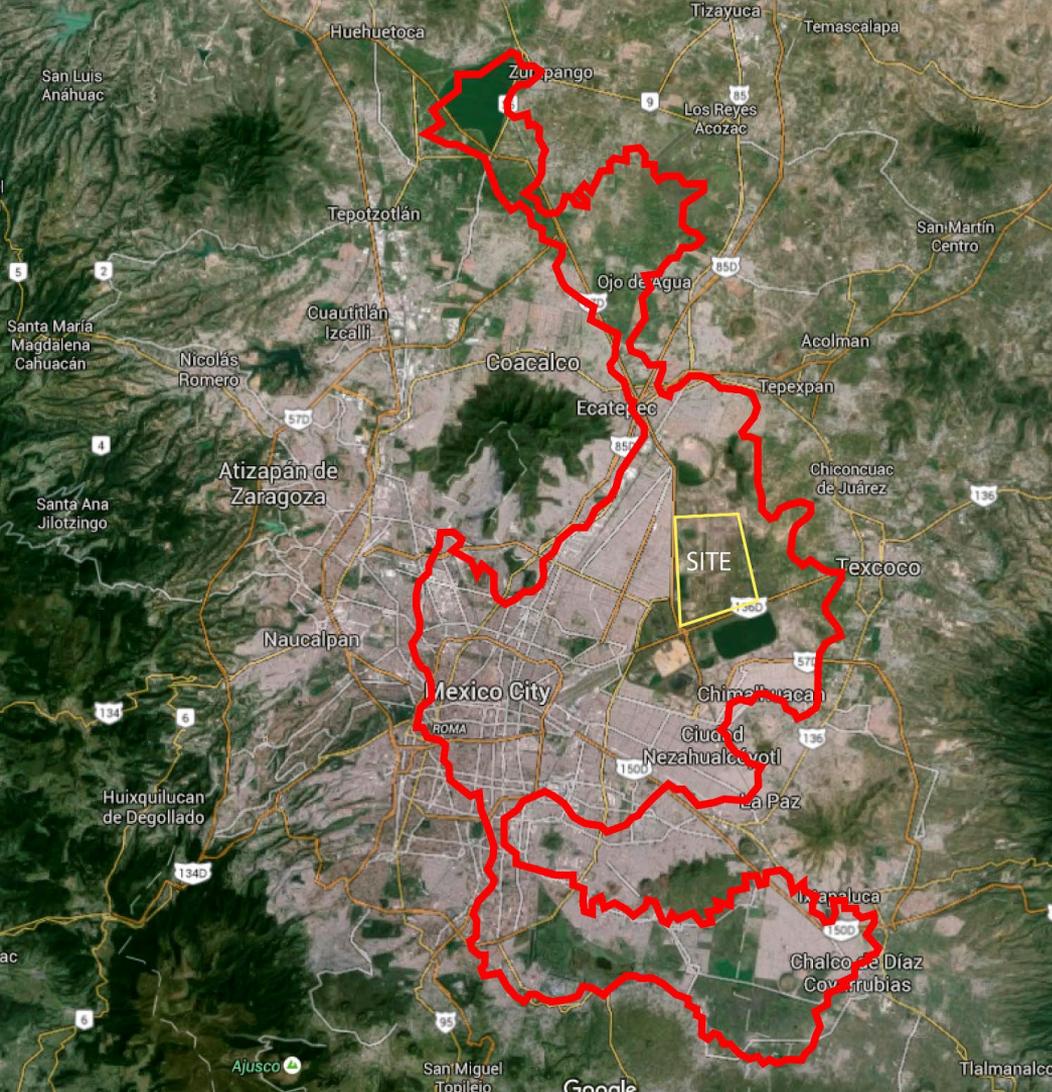


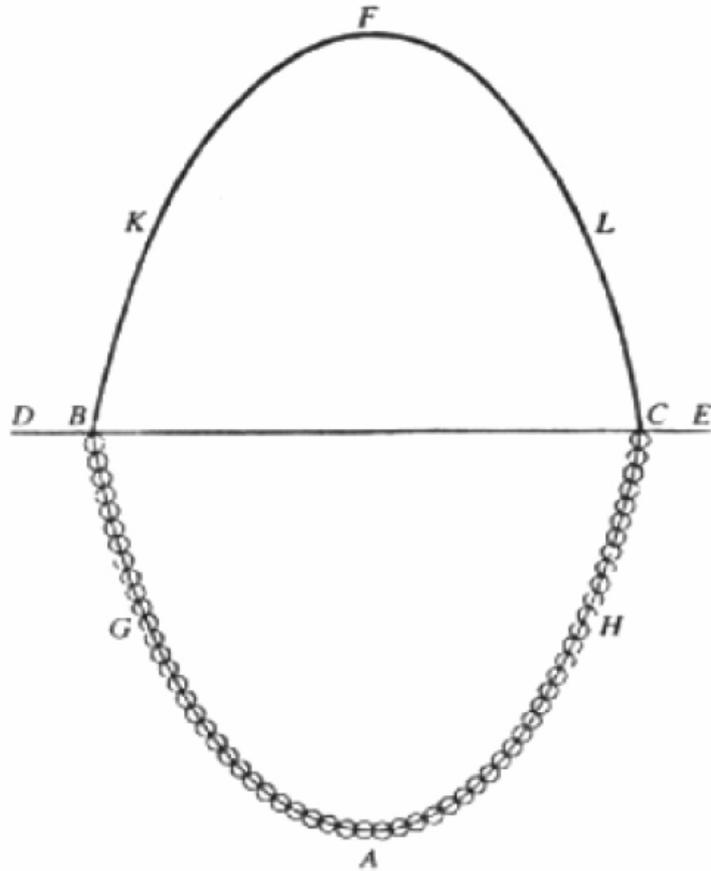
Figure 5. Final surface

Reference: The analytic and numerical definition of the geometry of the British Museum Great Court Roof.  
Williams, C. J. K. (2001)



“As hangs the flexible  
line, so but inverted will  
stand the rigid arch”

Robert Hooke 1675



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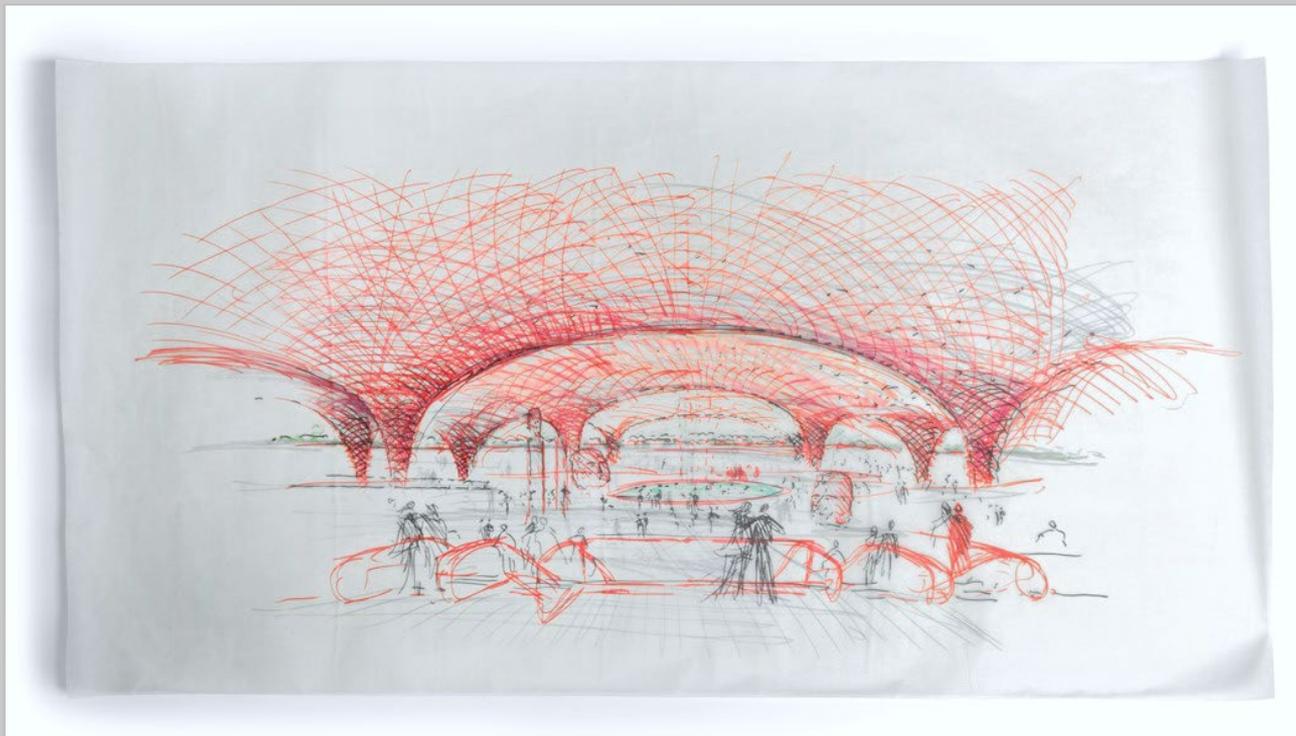


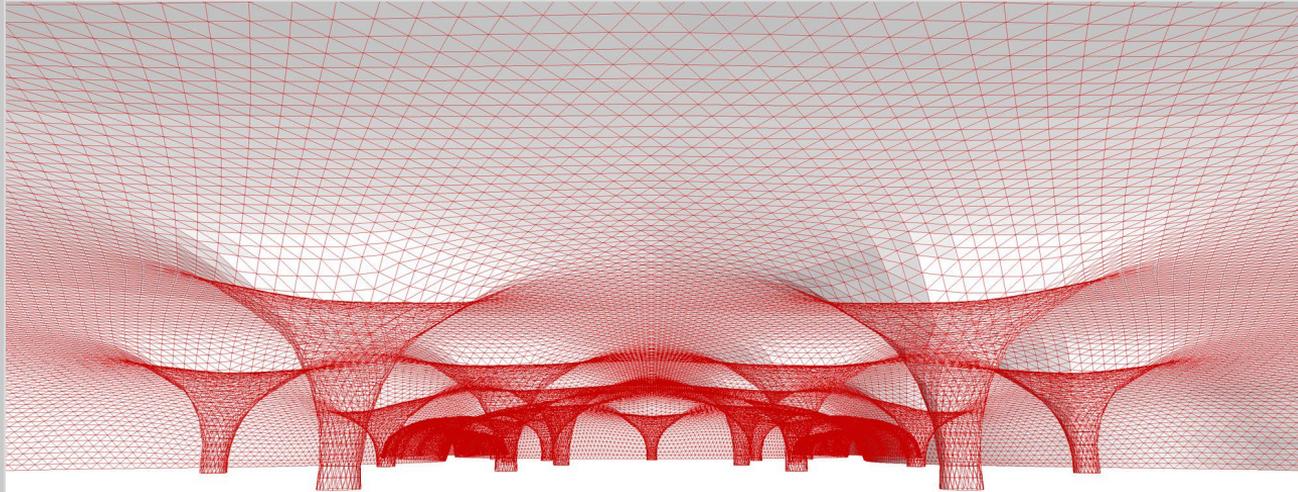


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Hanging chain concept model  
Photo: Foster + Partners





## Dynamic Relaxation

$$P_{ix} + \sum_{m=1}^n \frac{E_m A_m}{L_{mx}} (x_i - x_j) = M_{ix} a_{ix}.$$

$$a_{ix}^{(t+\Delta t)} = \frac{P_{ix} + \sum_{m=1}^n \frac{E_m A_m}{L_{mx}} (x_i^t - x_j^t)}{M_{ix}^t}.$$

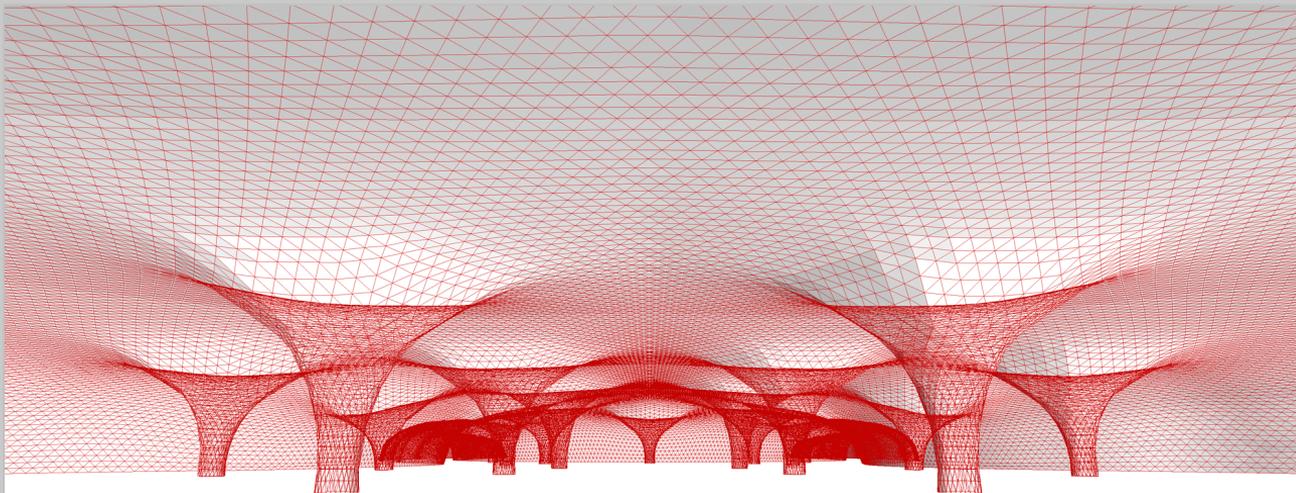
$$v_{ix}^{(t+\Delta t)} = C v_{ix}^t + a_{ix}^{(t+\Delta t)} \Delta t.$$

$$x_i^{(t+\Delta t)} = x_i^t + v_{ix}^{(t+\Delta t)} \Delta t.$$

Image: Foster + Partners (JO)



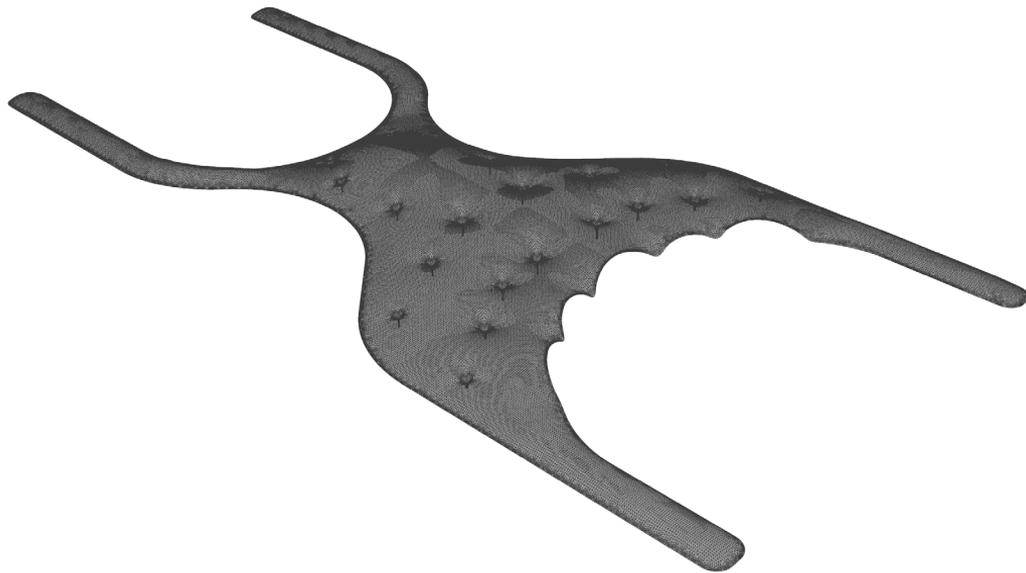
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Animation: Foster + Partners (JO)



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Animation: Foster + Partners (JO)



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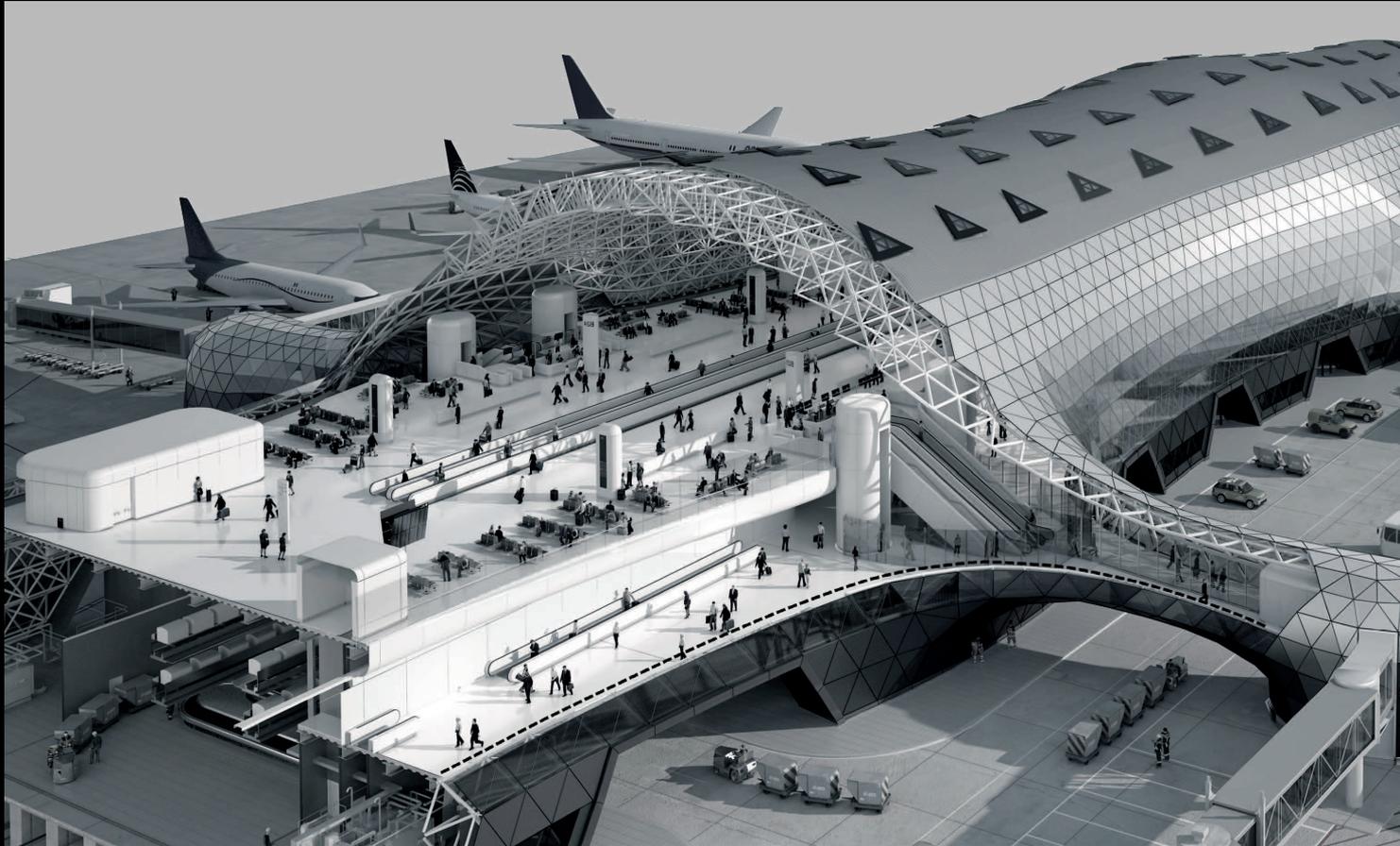


Render: Foster + Partners



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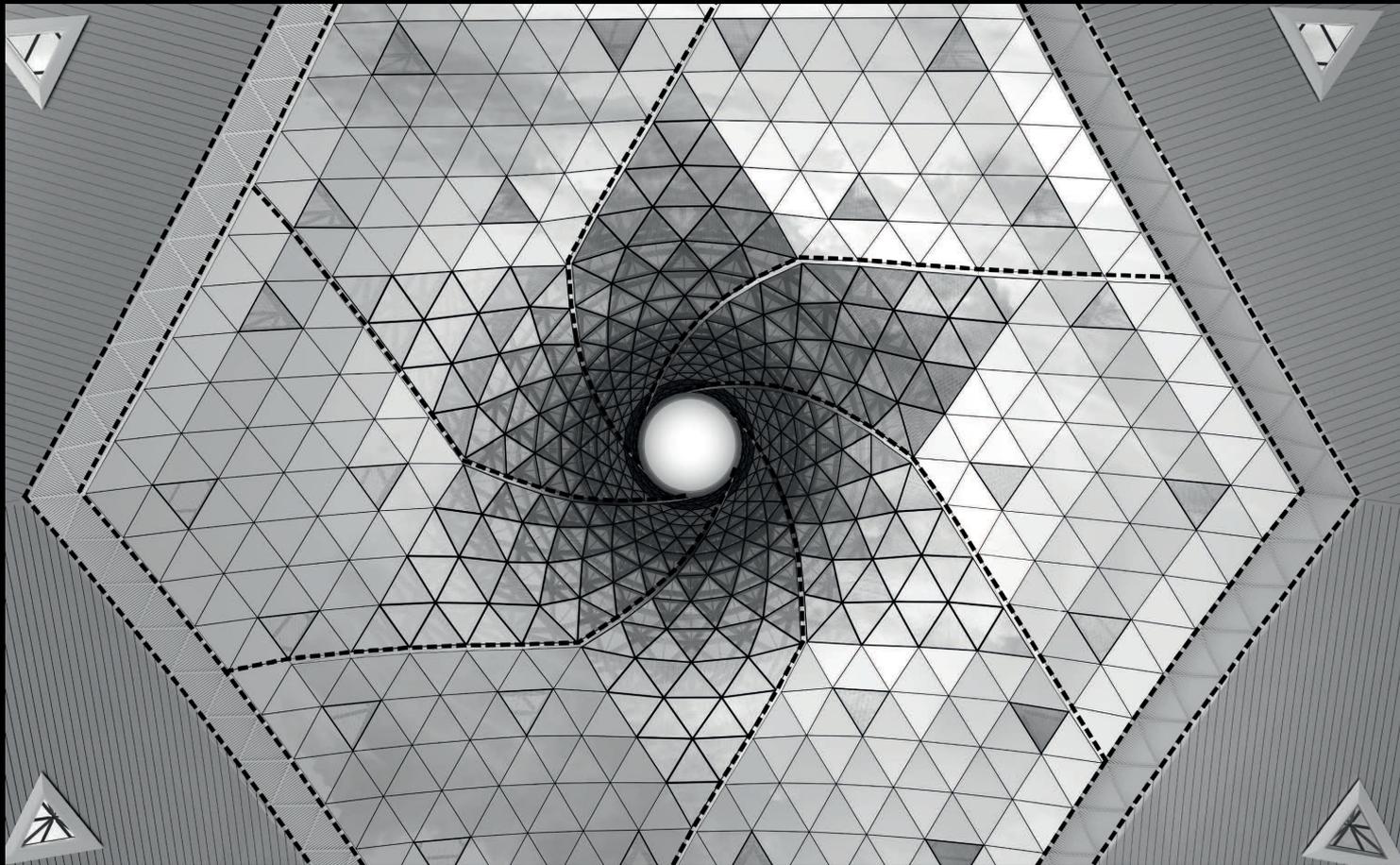




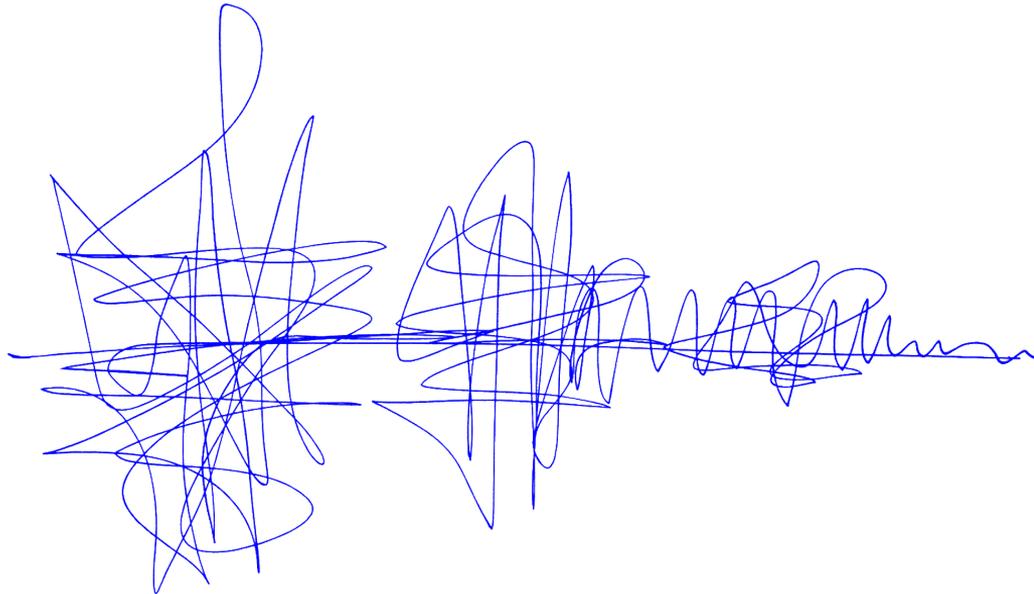
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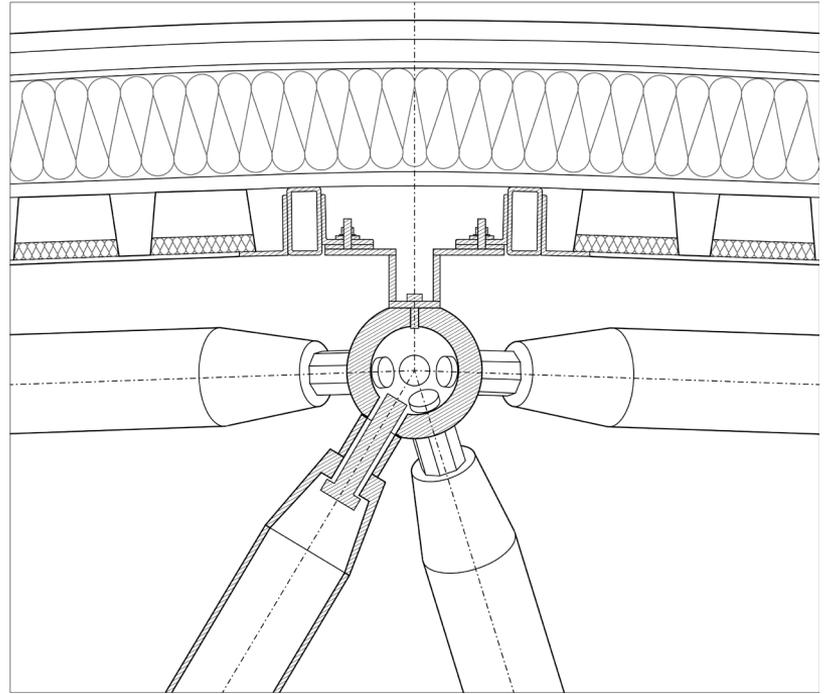
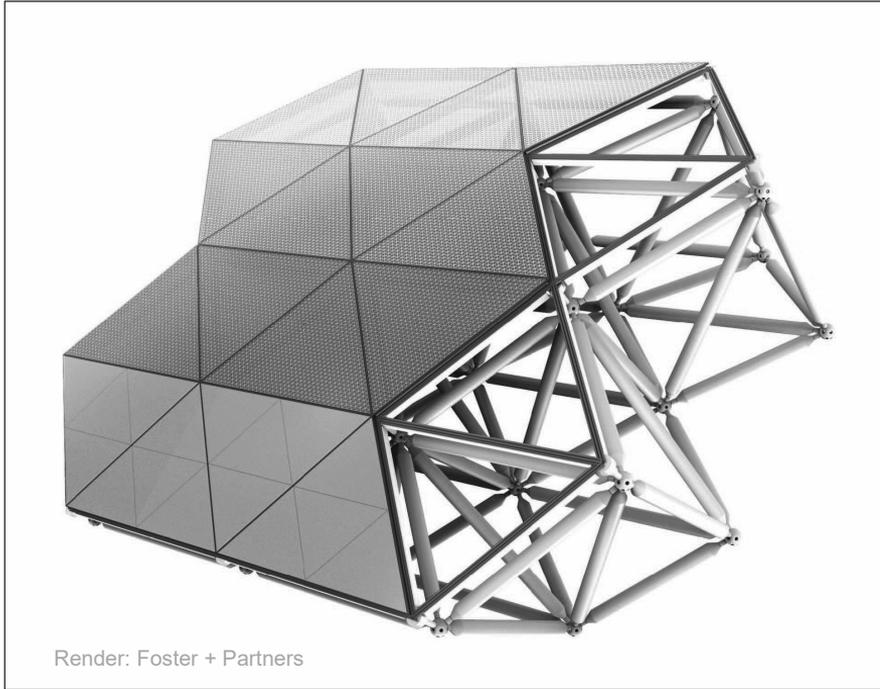
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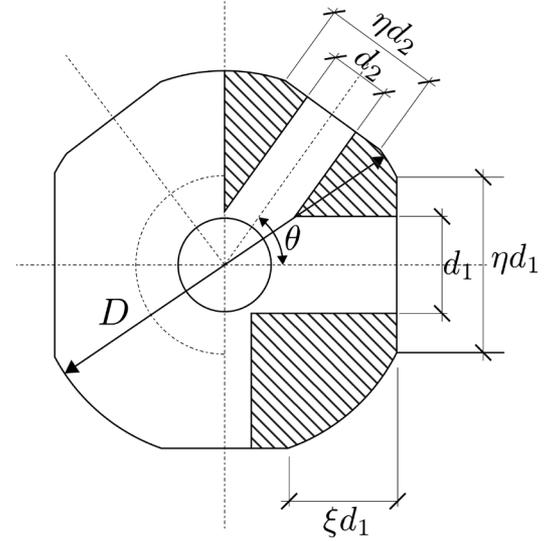
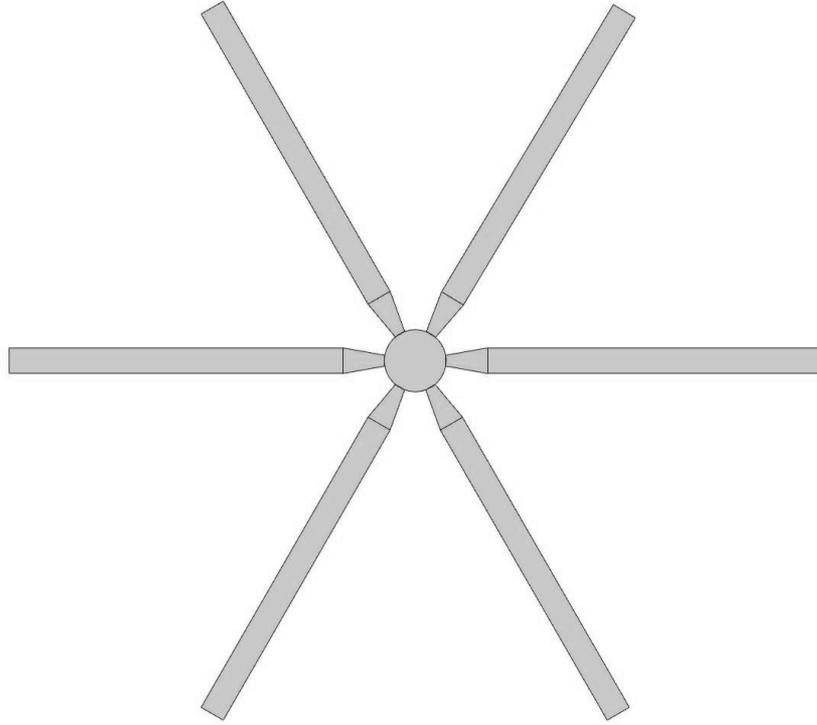






Time line and design development



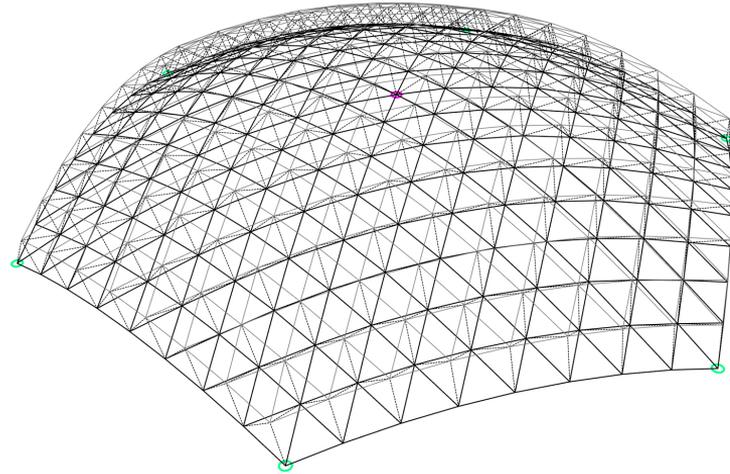


$$D \geq \sqrt{\left(\frac{d_2}{\sin(\theta)} + (d_1 \cot(\theta) + 2\xi d_1)\right)^2 + \eta^2 d_1^2}$$

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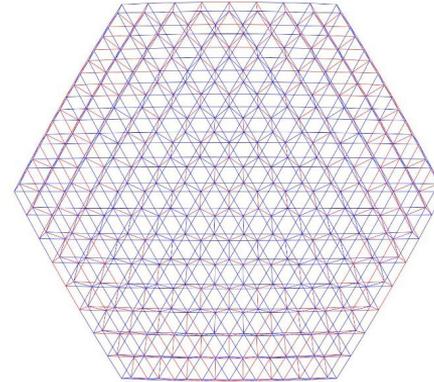
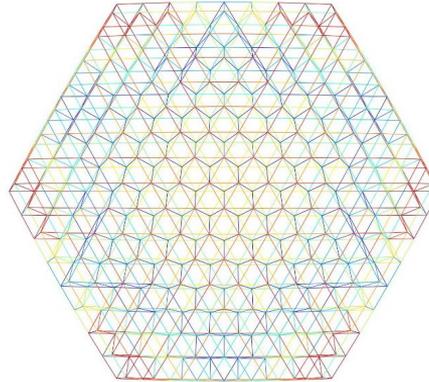
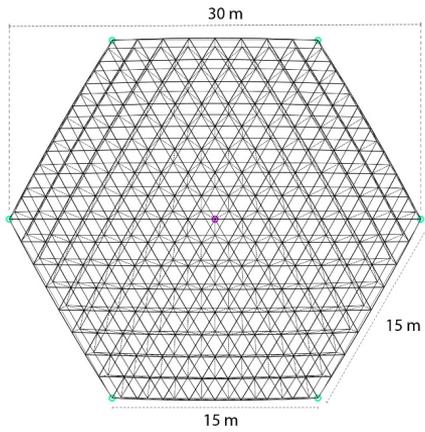
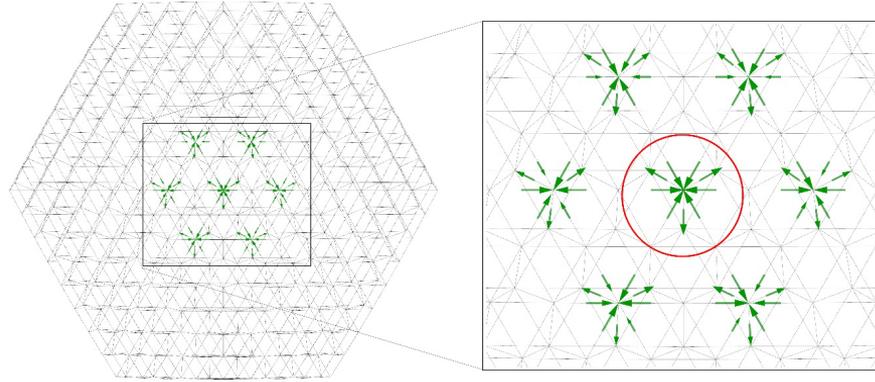
# Nodal form finding

Design of material-efficient nodal connection for a tetrahedral space frame.





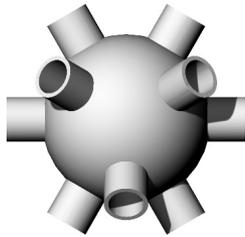
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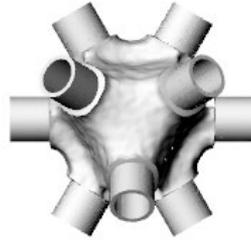


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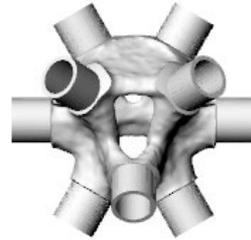
Starting geometry



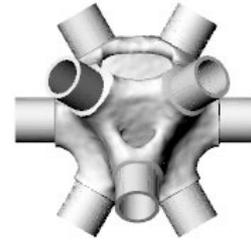
Symmetric load



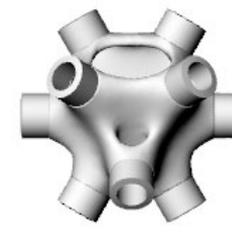
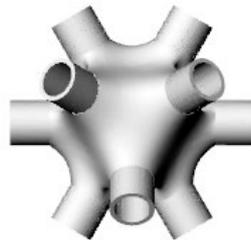
Asymmetric load 1



Asymmetric load 2



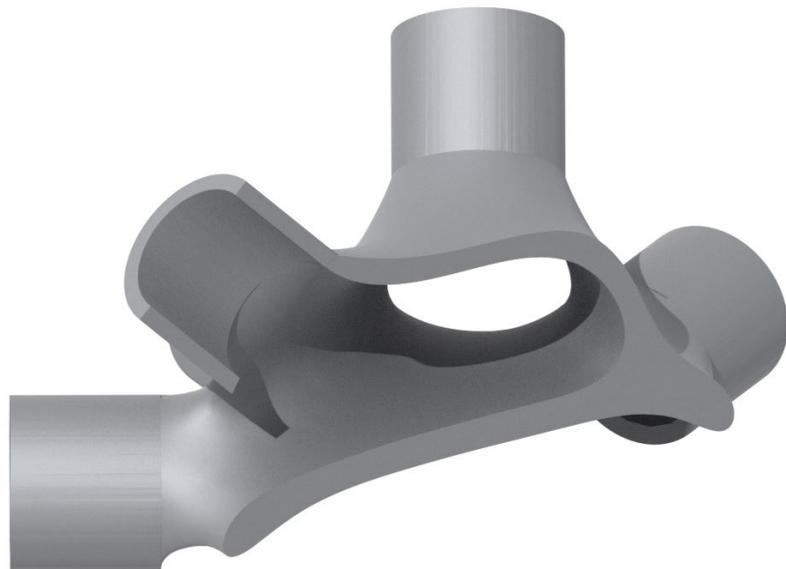
Raw TO result



Postprocessed geometry

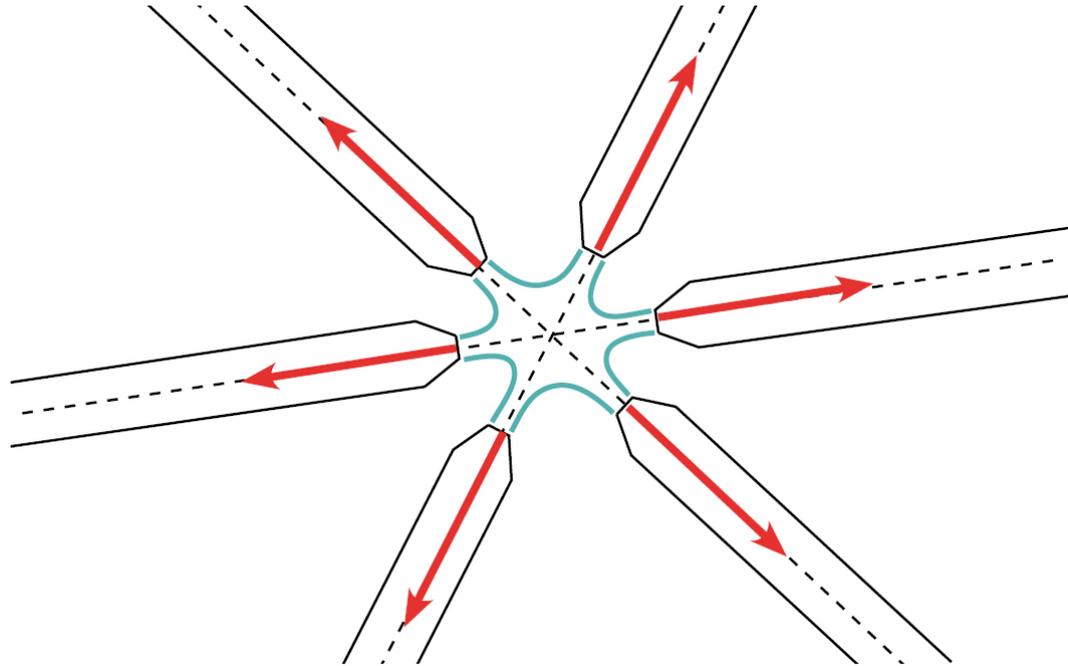


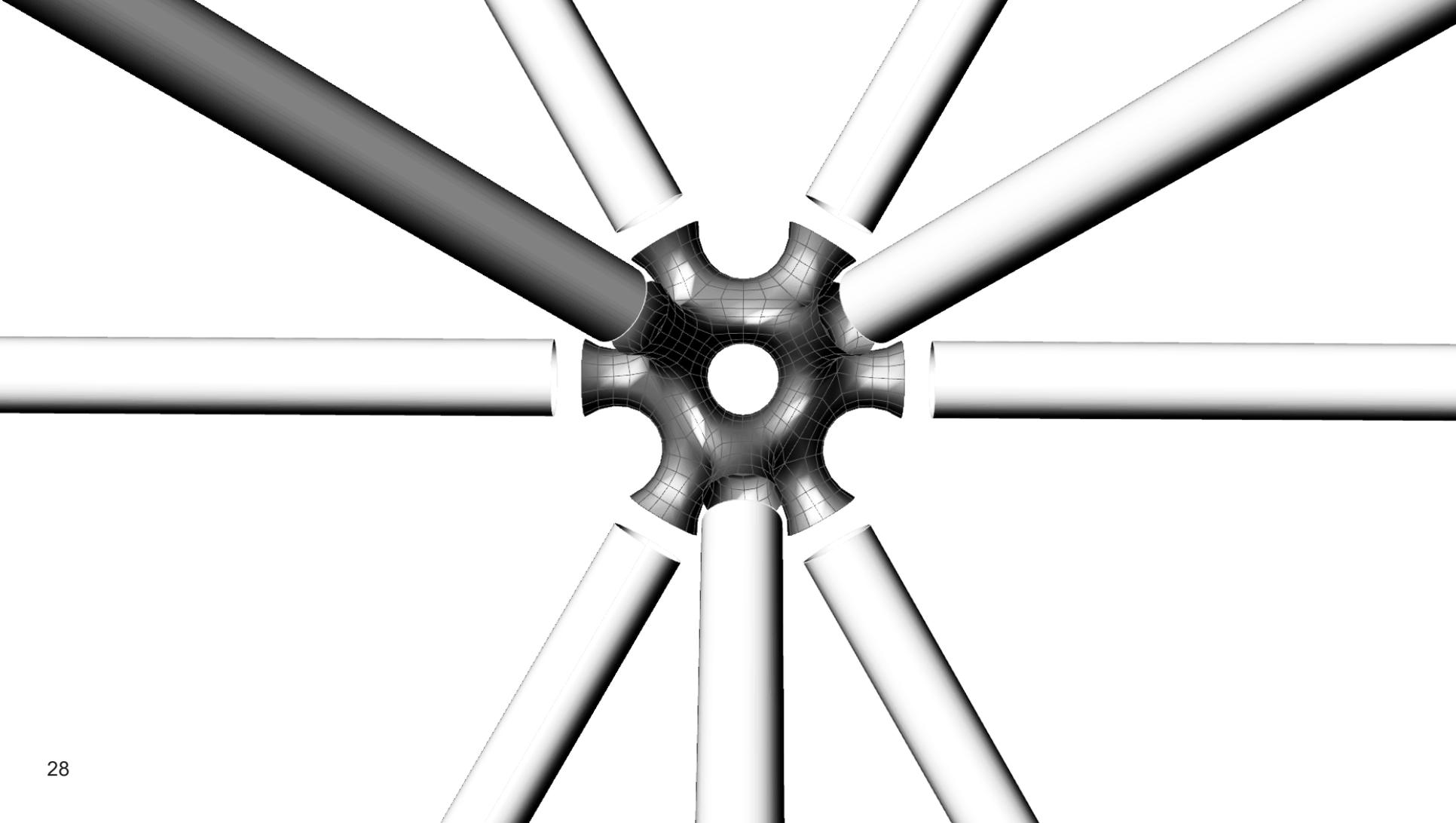
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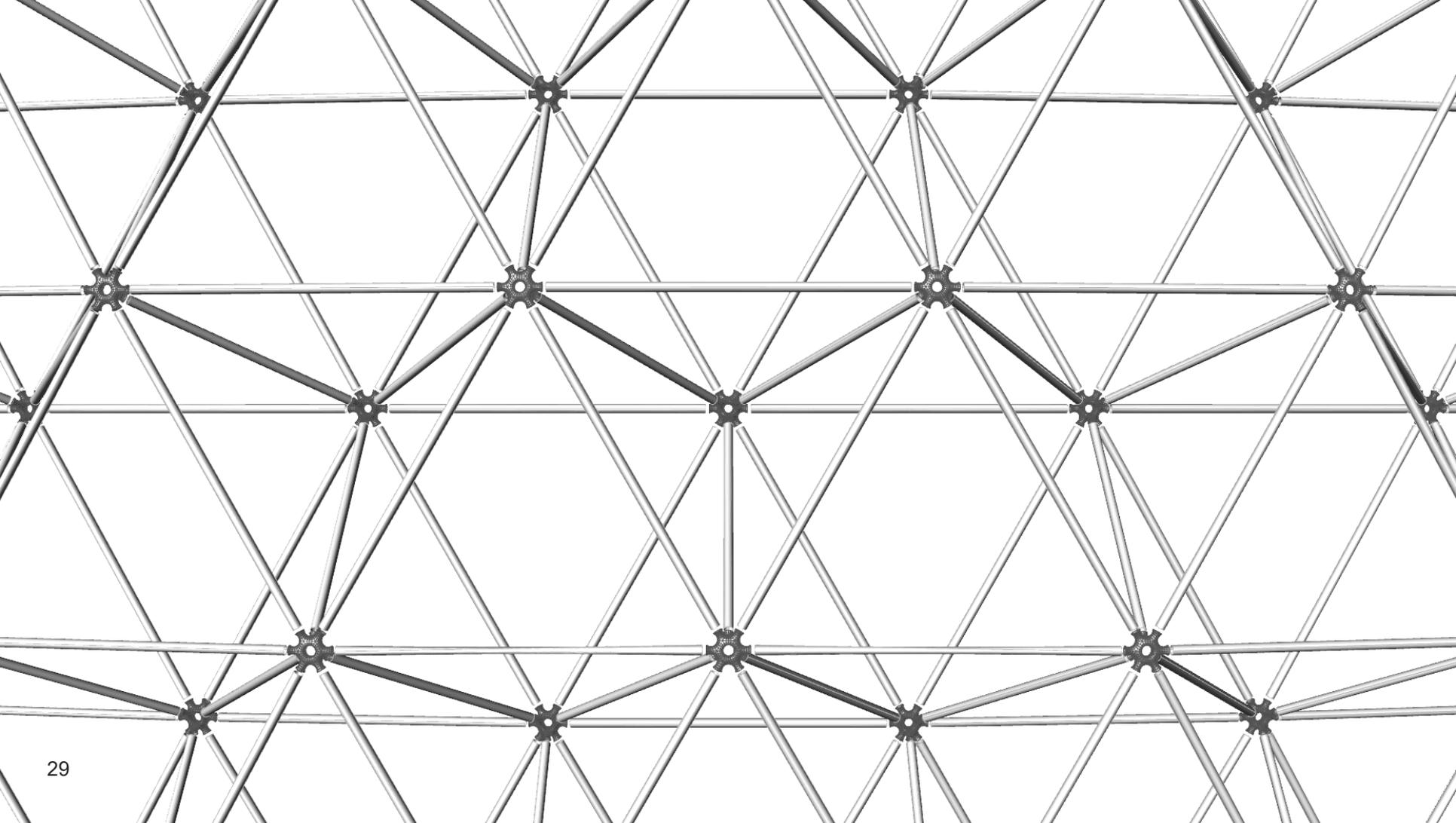


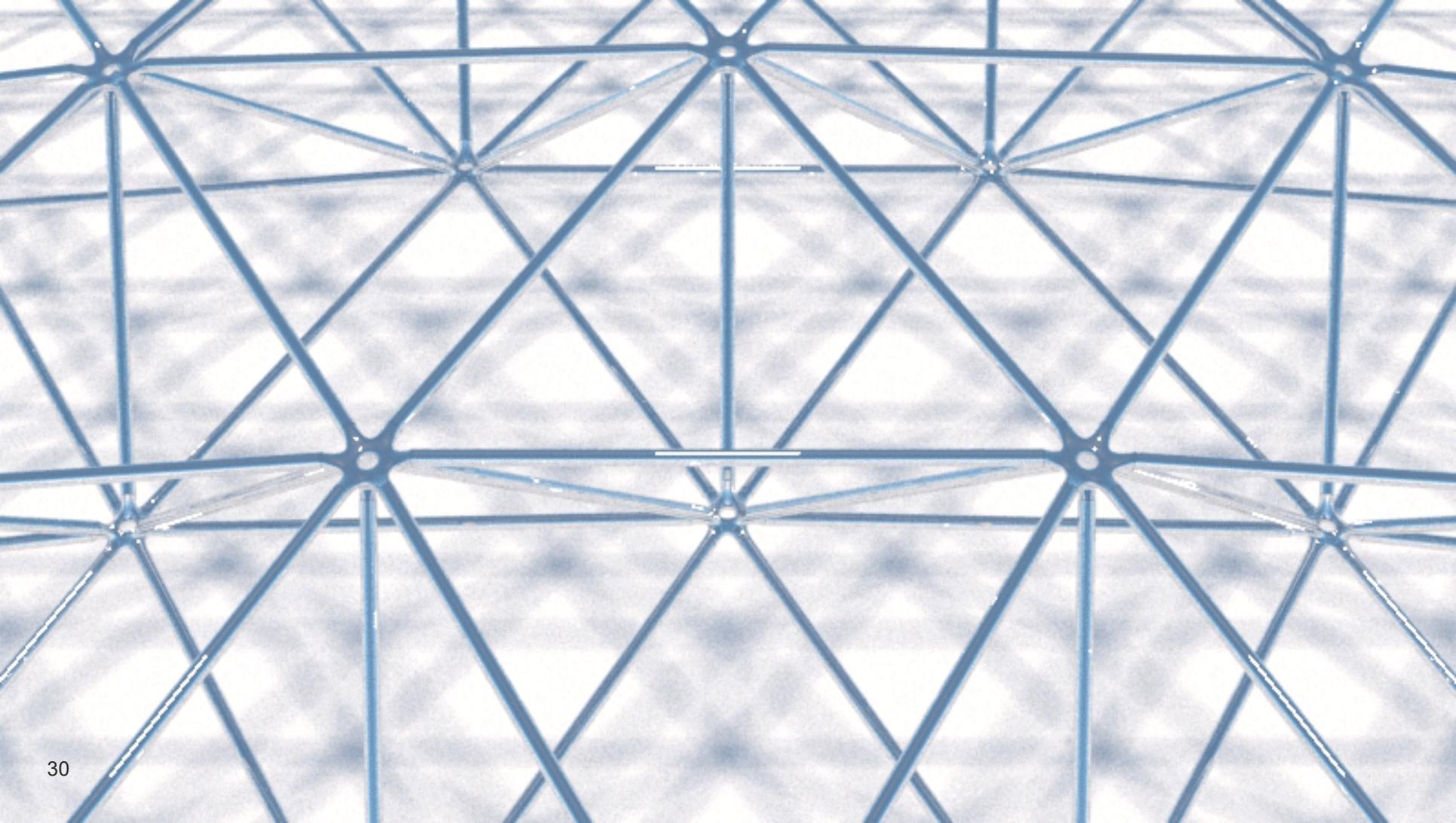


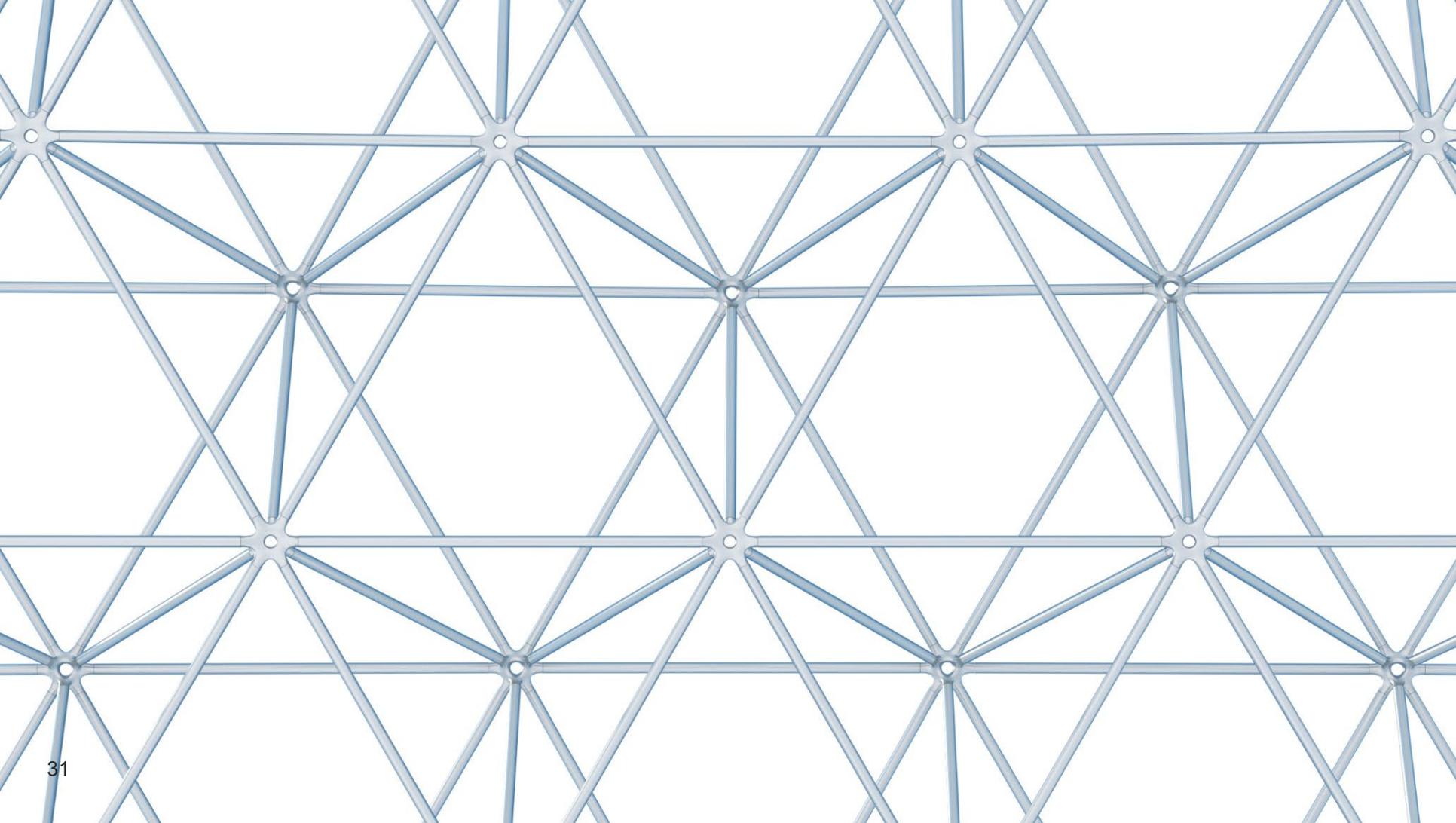
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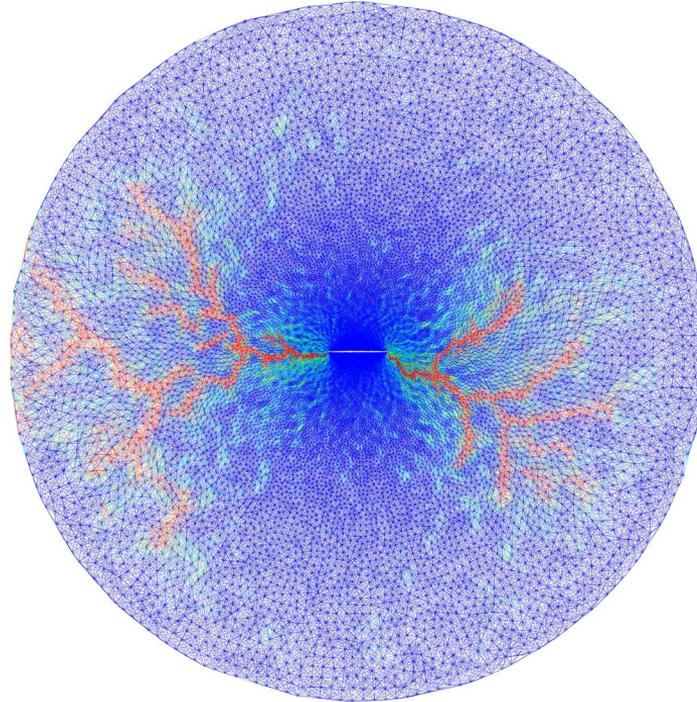






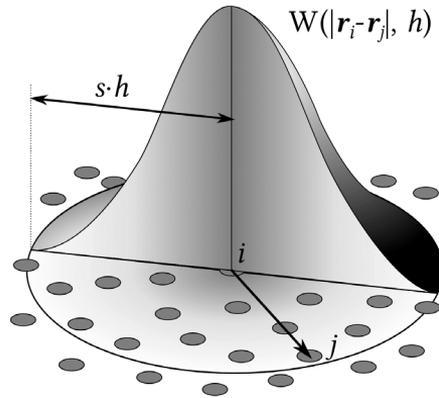


# Fracture and yielding

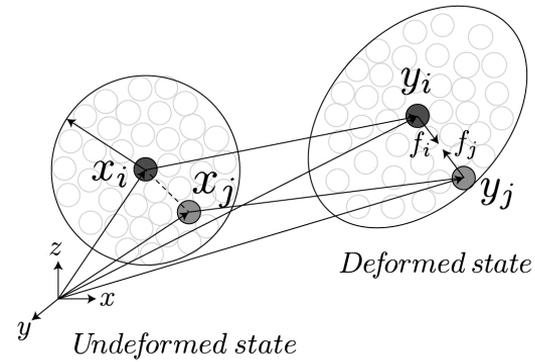


# Meshless methods

Smoothed Particle Hydrodynamics (SPH)



Peridynamics (PD)

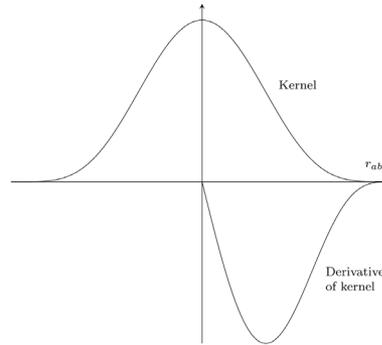


Newtons second law states:

$$\mathbf{F} = m\mathbf{a}$$

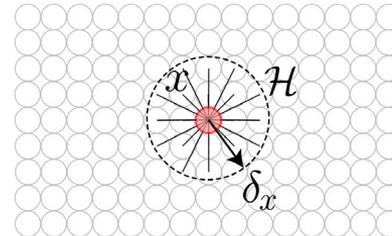
The differential form:

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x}, t) = \nabla \cdot \boldsymbol{\sigma} + \mathbf{b}(\mathbf{x}, t),$$



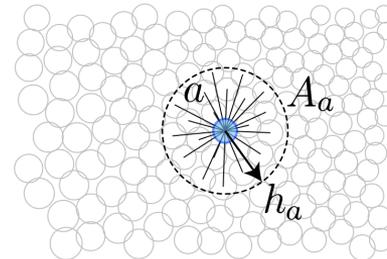
In peridynamics:

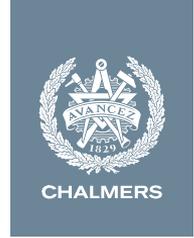
$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x}, t) = \int_{\mathcal{H}_x} \mathbf{f}(\mathbf{x}, \mathbf{x}', t) dV_{x'} + \mathbf{b}(\mathbf{x}, t),$$



In Smoothed particle hydrodynamics:

$$\frac{d\mathbf{v}_a}{dt} = \sum_b m_b \left( \frac{\sigma_a}{\rho_a^2} + \frac{\sigma_b}{\rho_b^2} \right) \nabla W(\mathbf{r}_a - \mathbf{r}_b, h_a) + \mathbf{g},$$



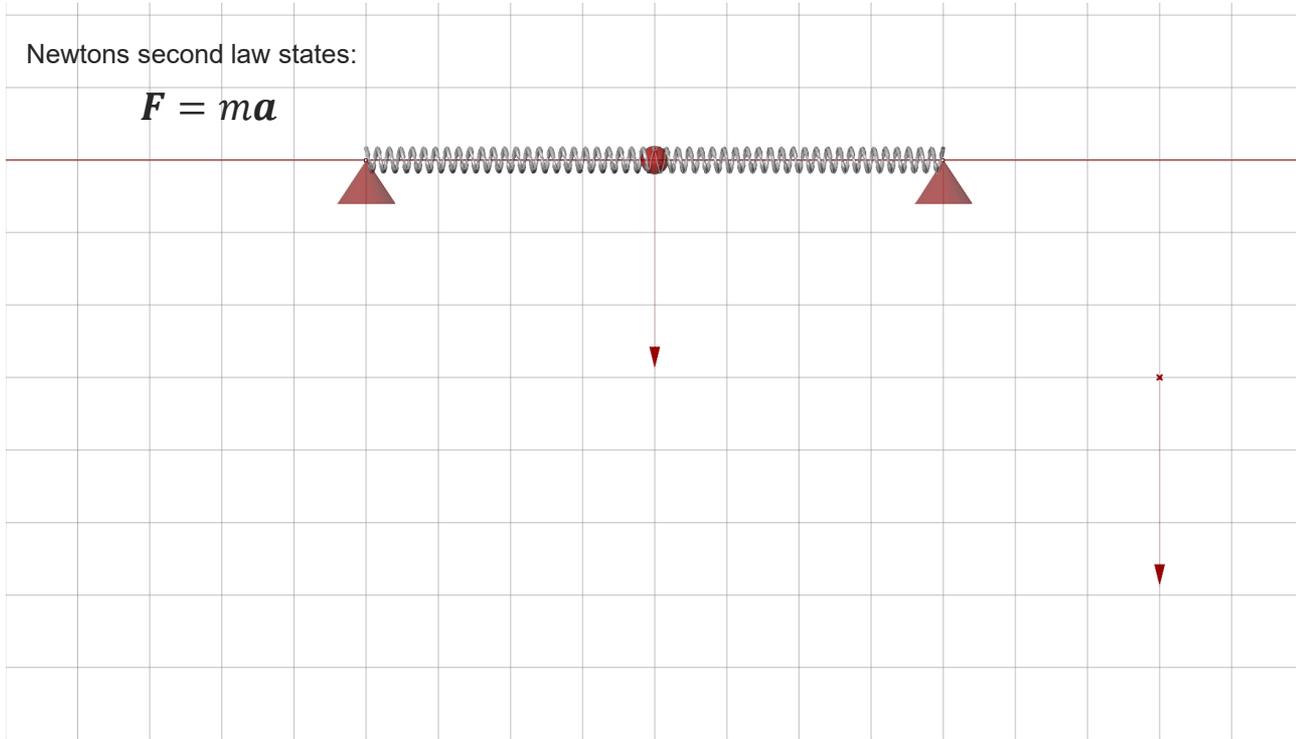


Newtons second law states:

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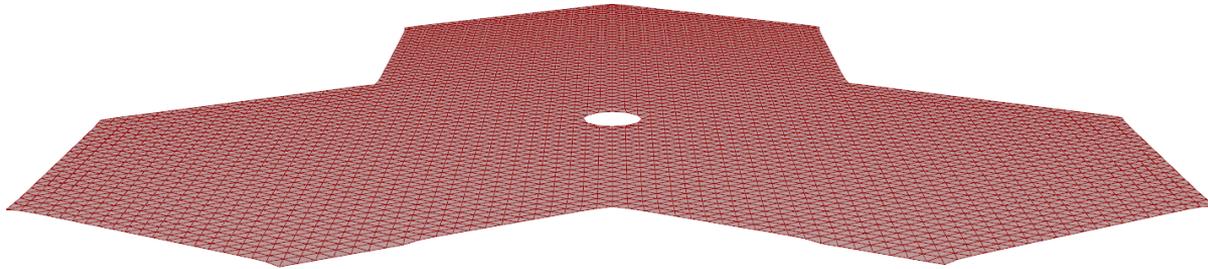
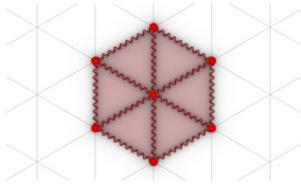
Newtons second law states:

$$F = ma$$



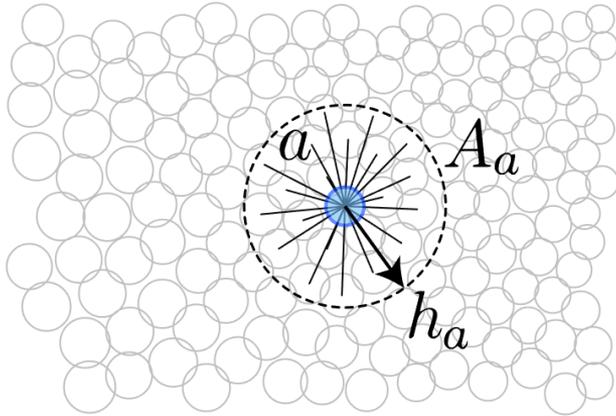
Newtons second law states:

$$\mathbf{F} = m\mathbf{a}$$



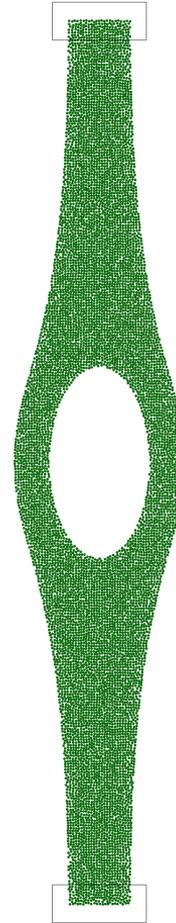
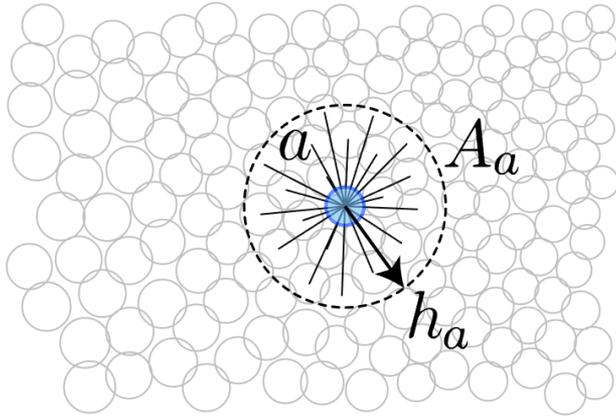
Newtons second law states:

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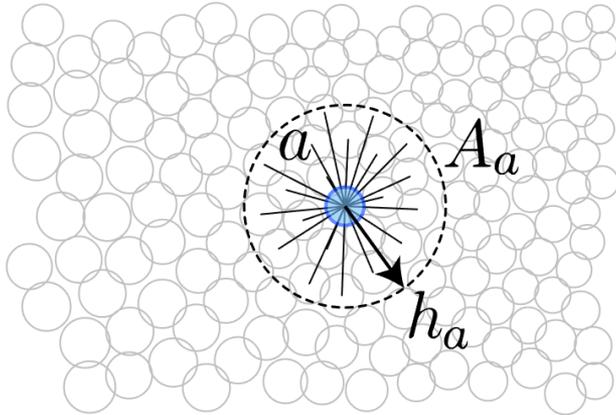
Newtons second law states:

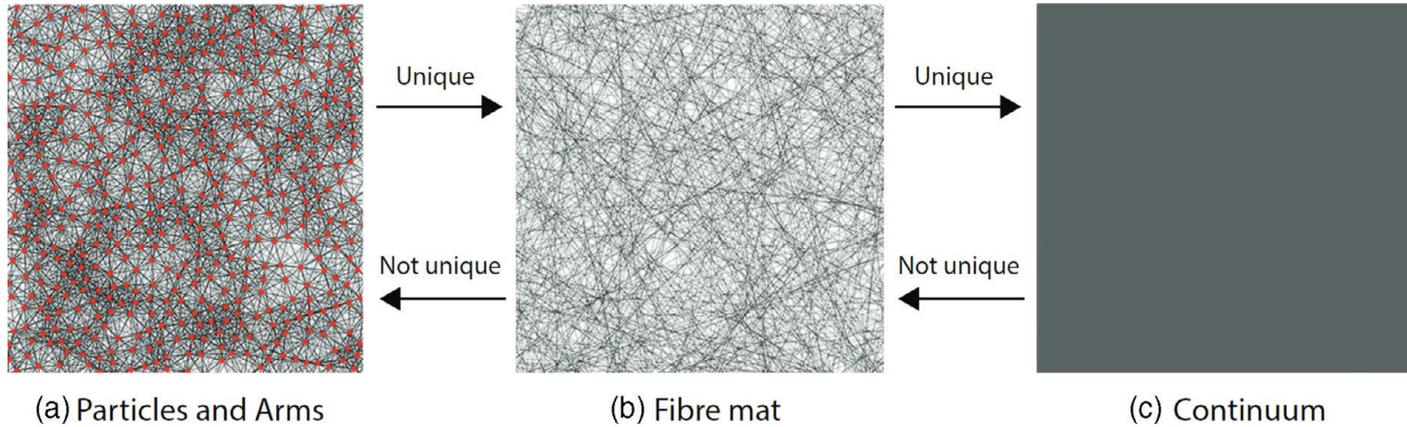
$$\mathbf{F} = m\mathbf{a}$$

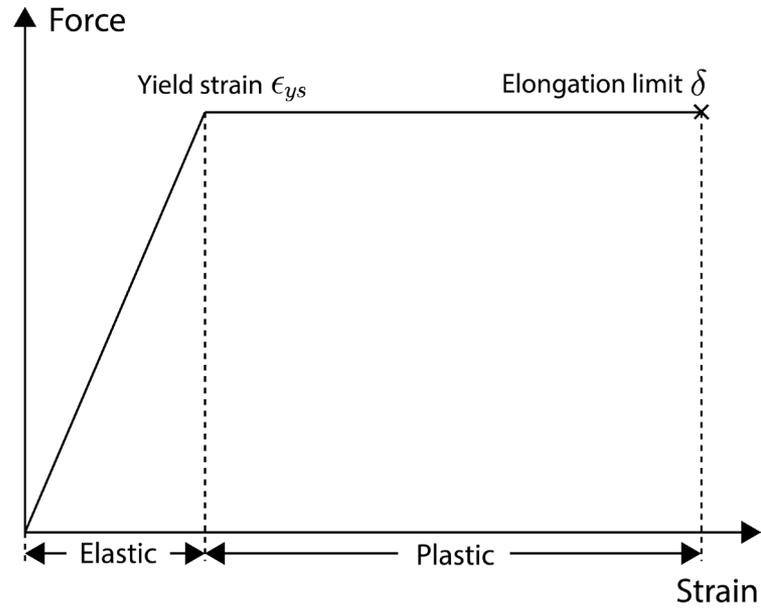


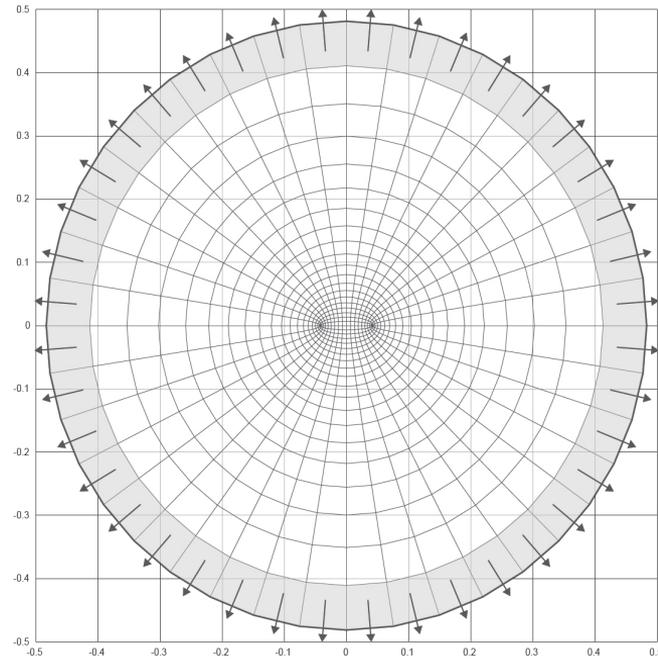
Newtons second law states:

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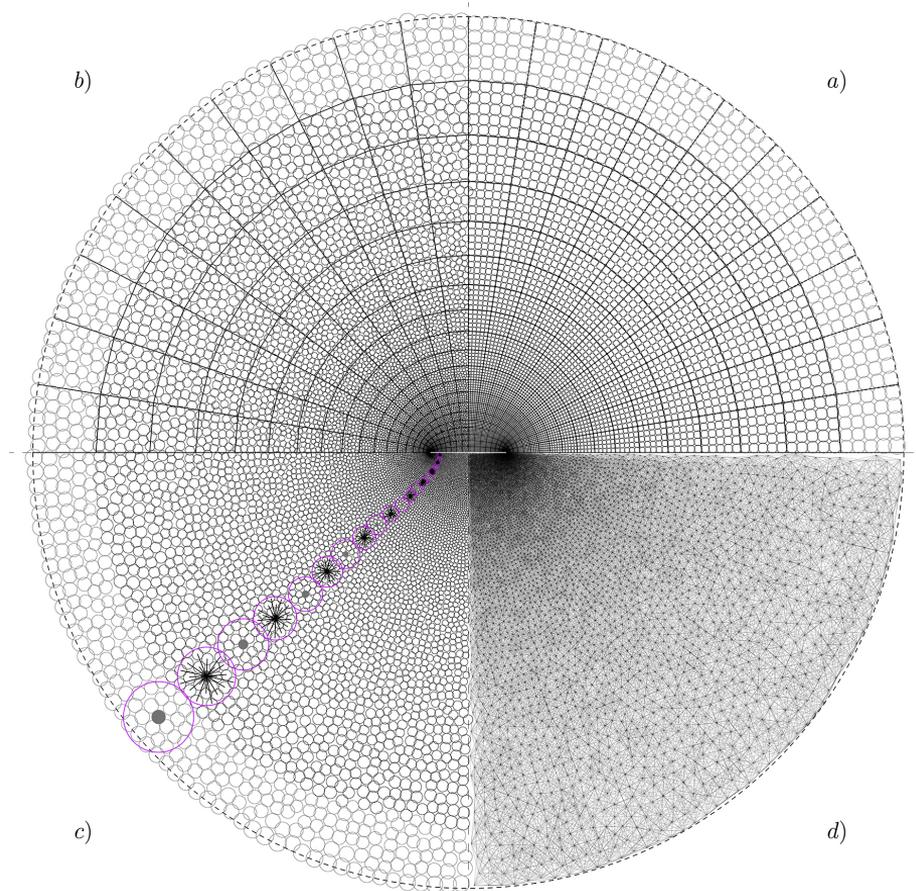




$$x = c \cosh(\xi) \cos(\eta),$$
$$y = c \sinh(\xi) \sin(\eta).$$

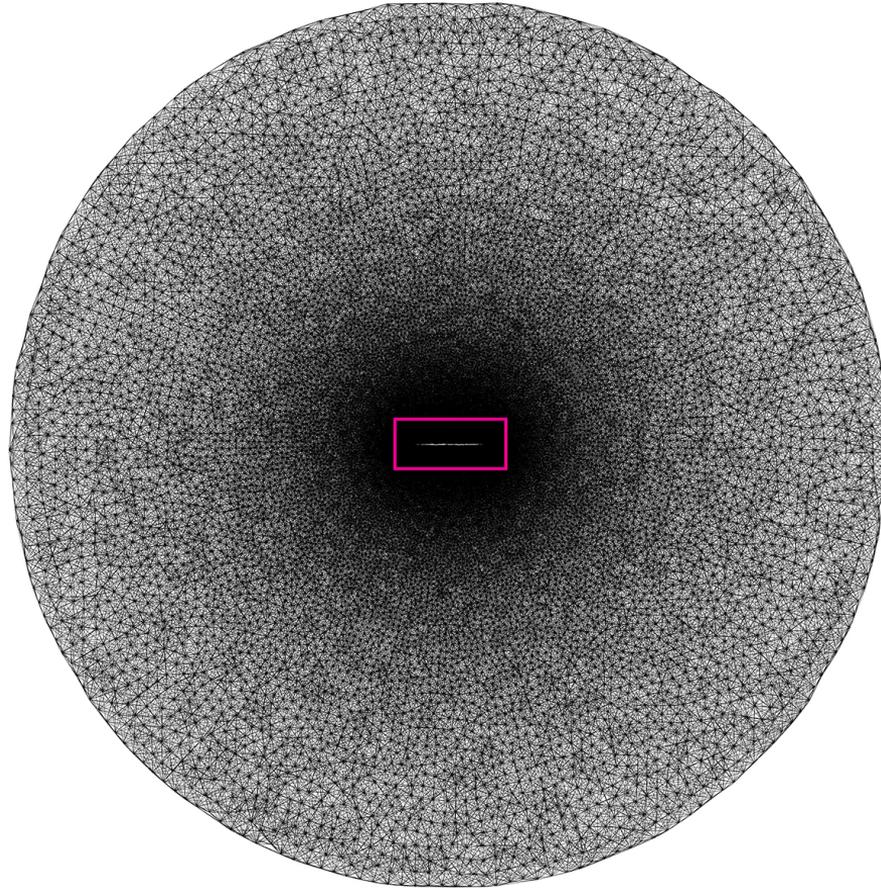


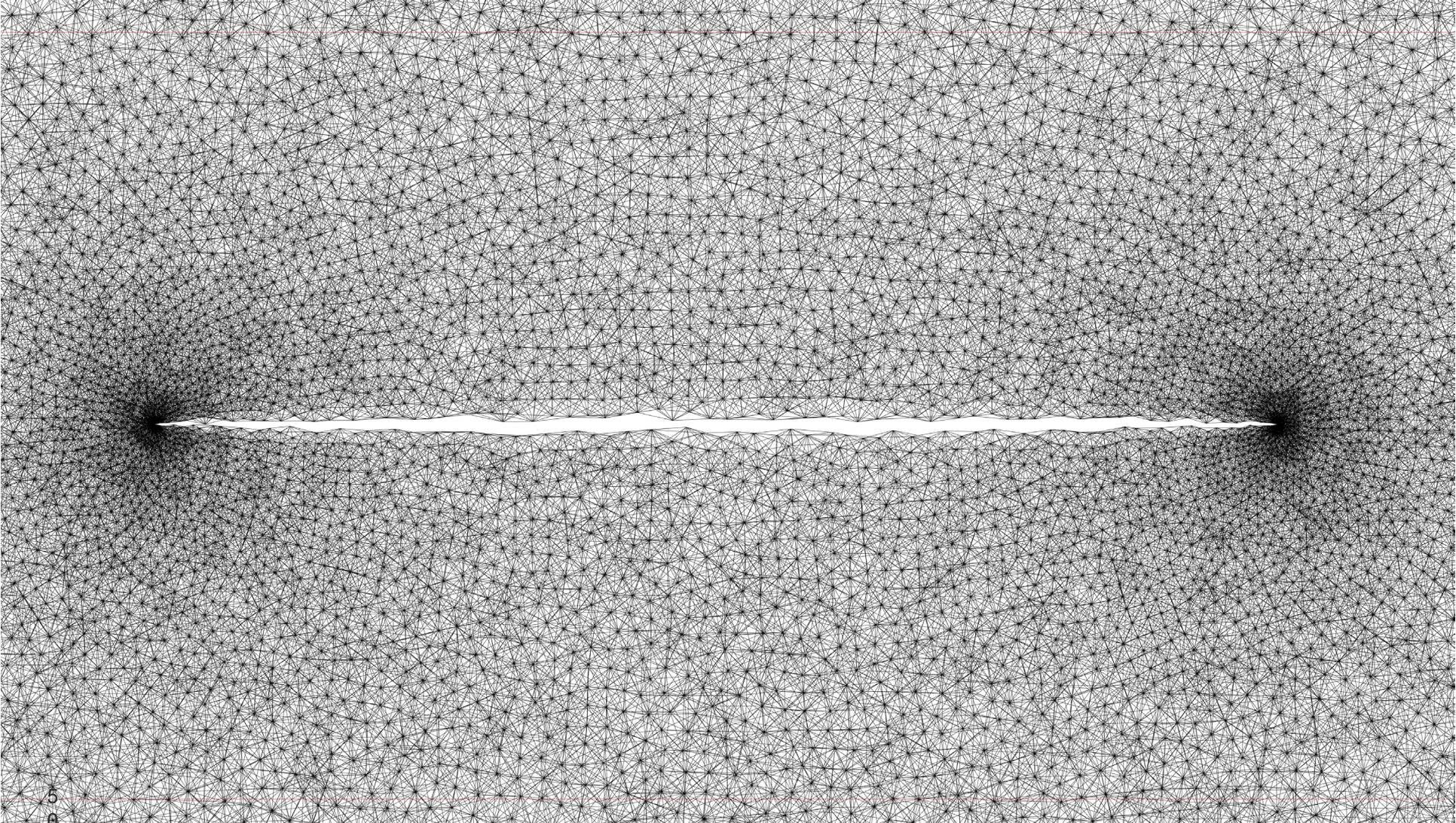
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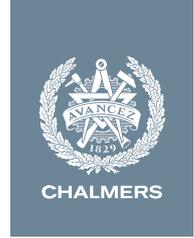


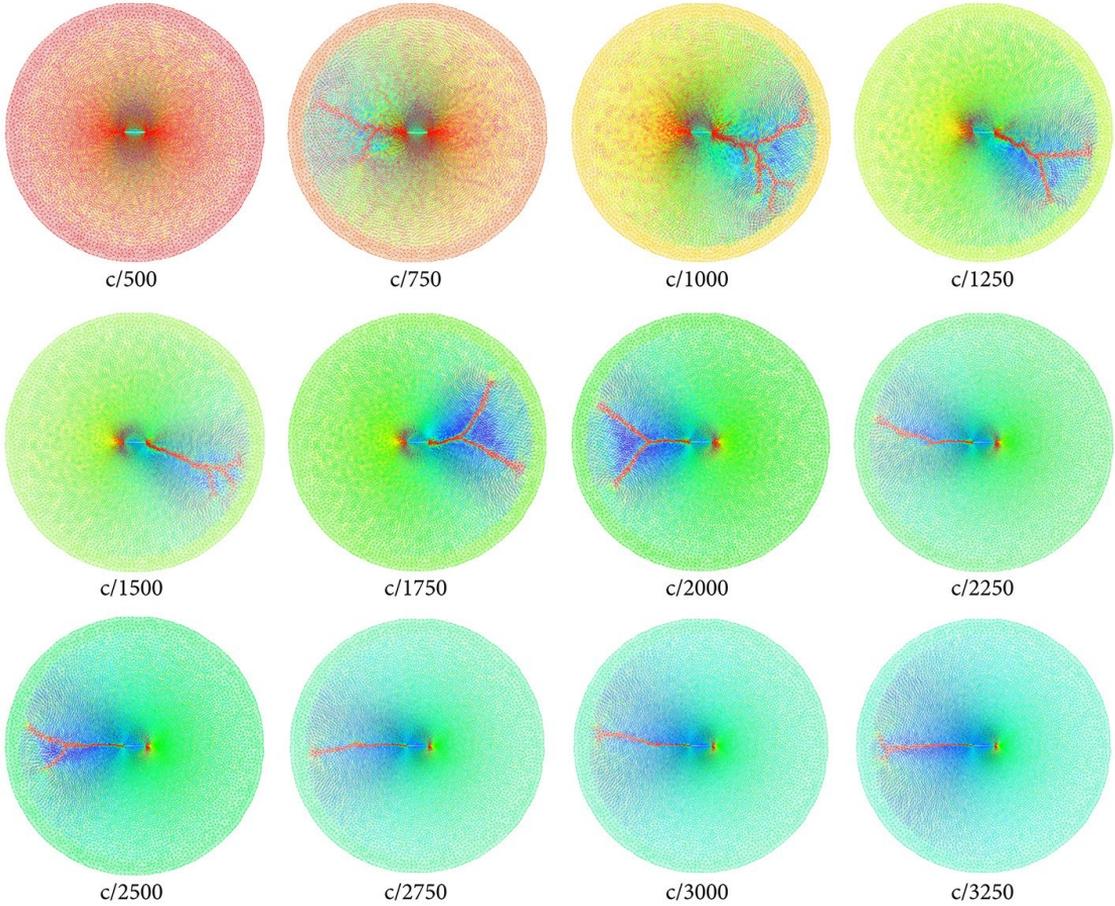


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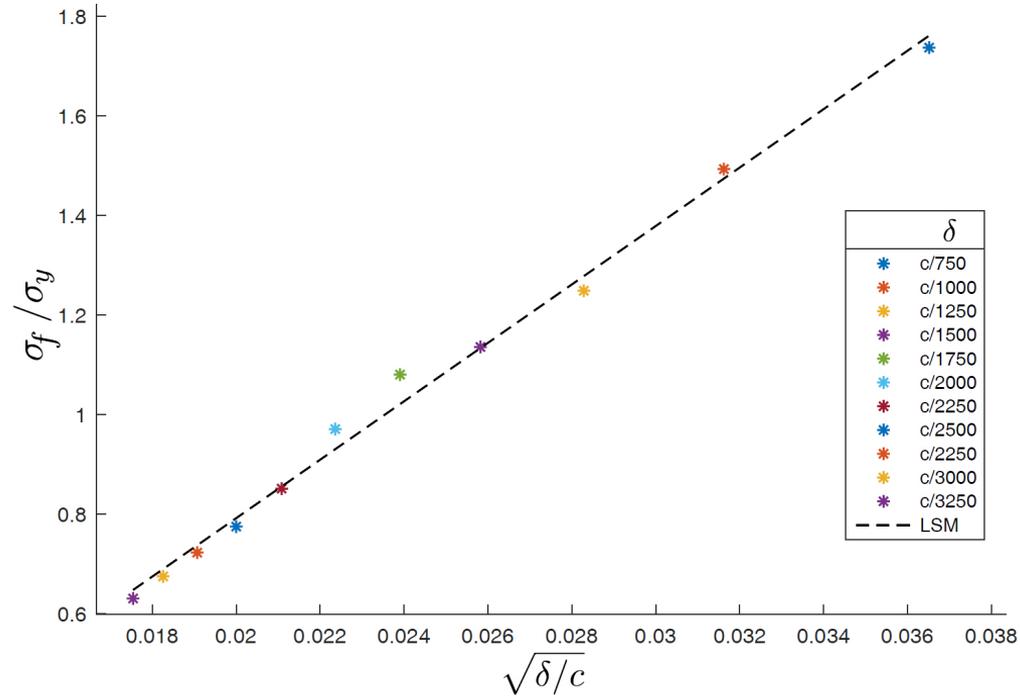








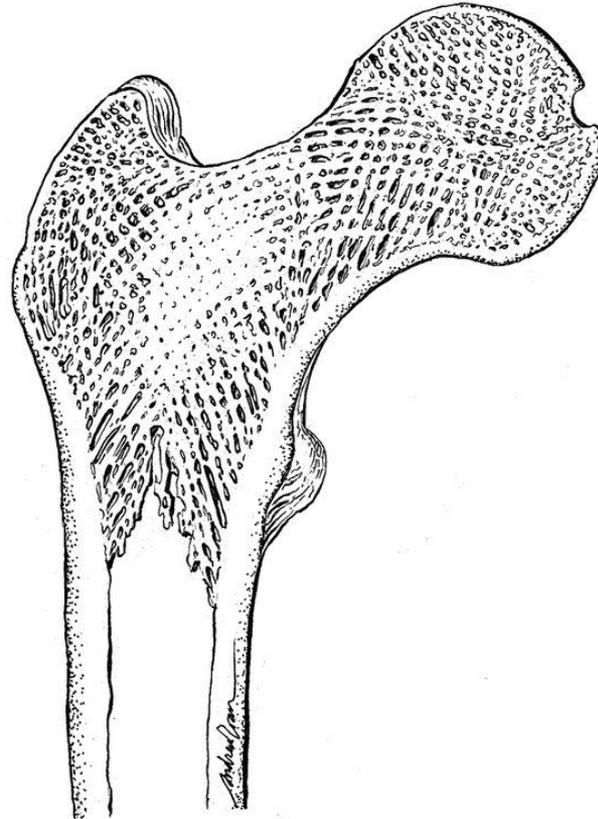
Fracture stress  $\sigma_f$  compared with elongation limit  $\delta$



# Porous structures

*“In a mature bone where the general form is established, the bone elements place or displace themselves, and decrease or increase their mass, in response to the mechanical demands imposed on them.”*

- Wolff (1892)



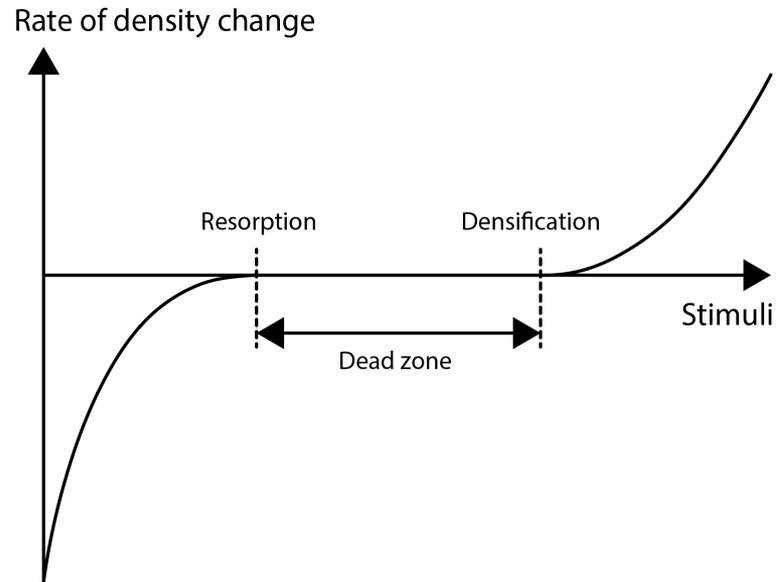
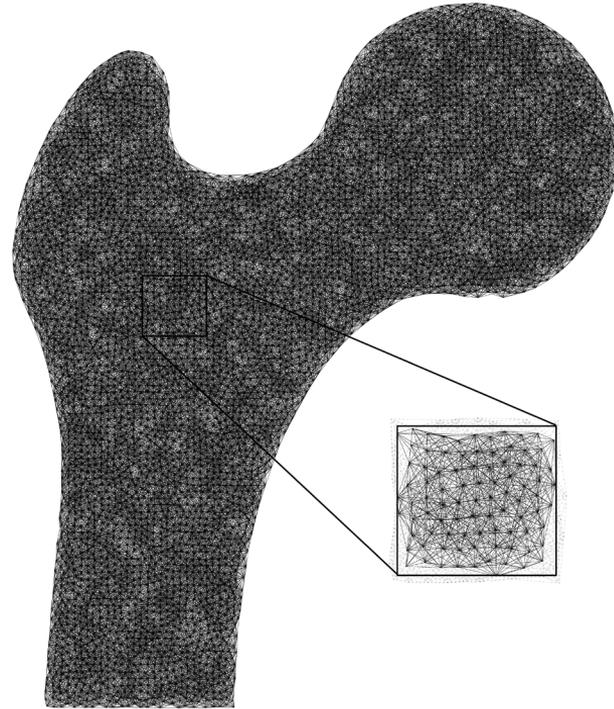
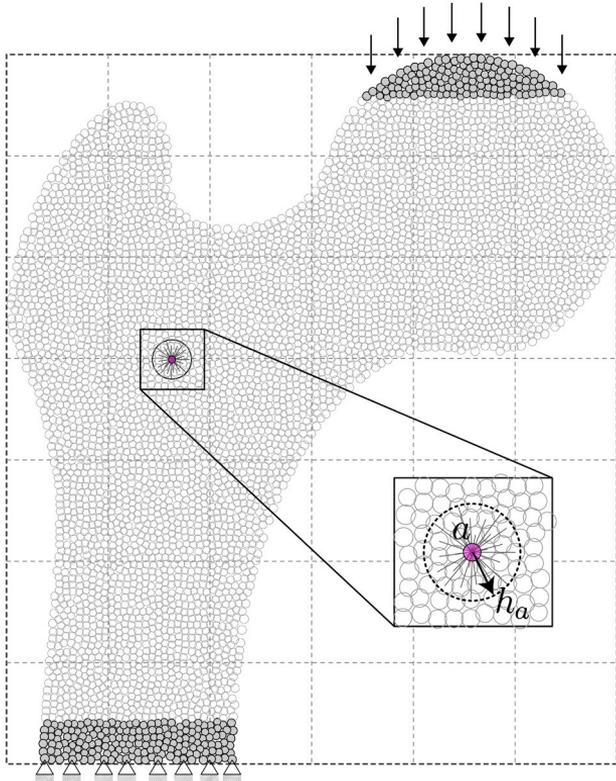
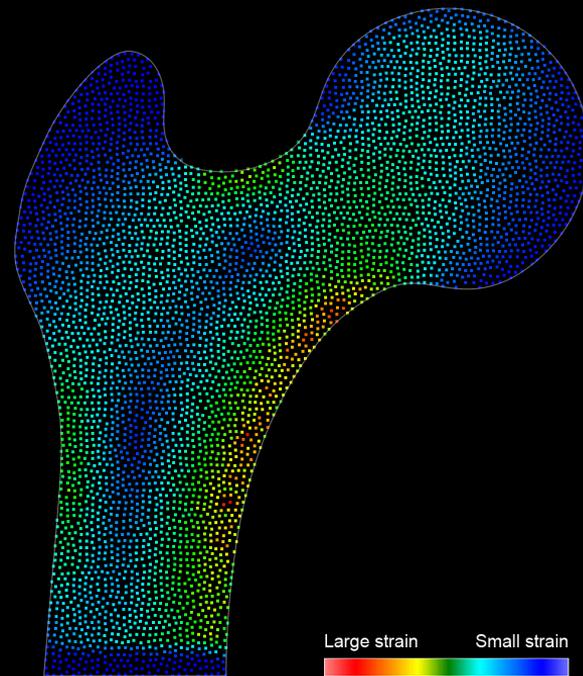
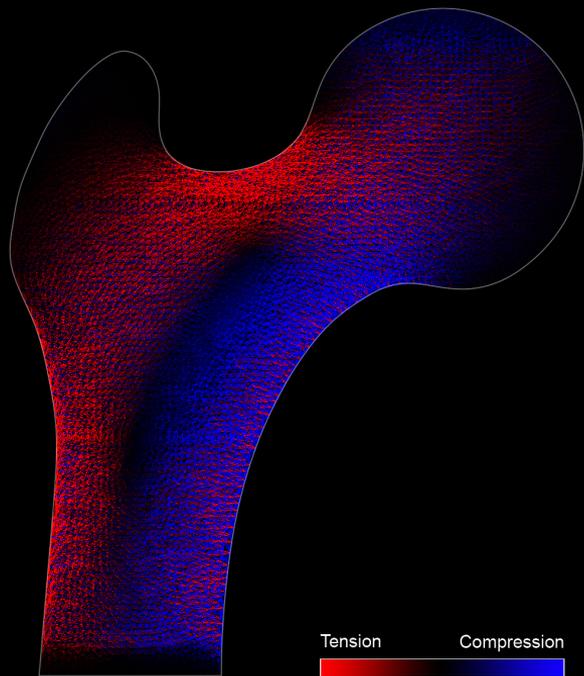


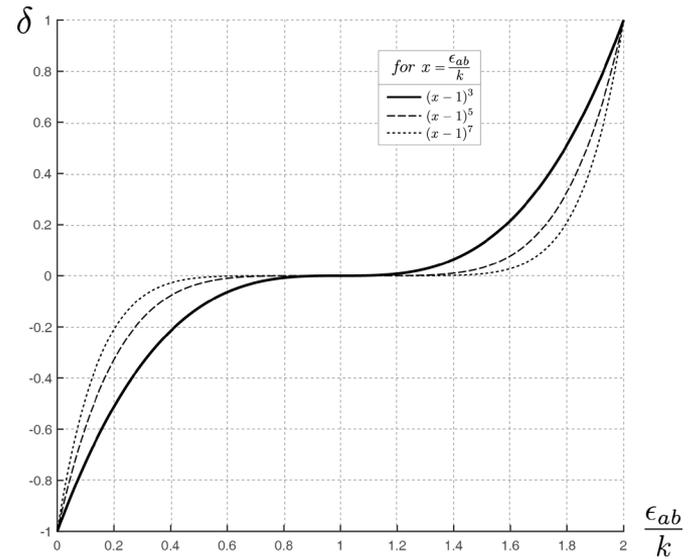
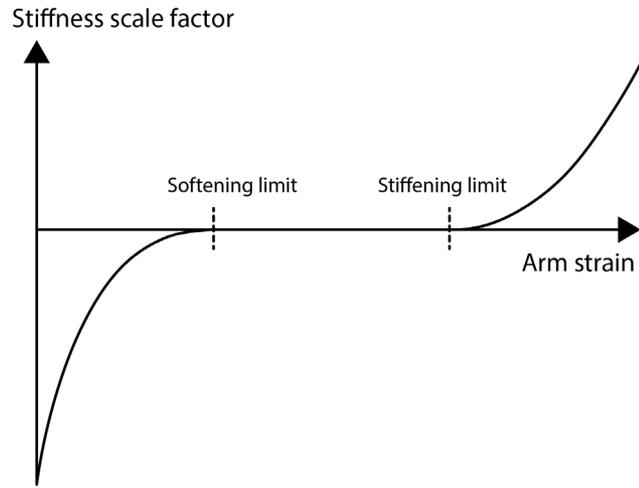
Illustration of the Mechanostats theory by Frost from the 60's. Reference: *Osteocyte-viability-based simulations of trabecular bone loss and recovery in disuse and reloading*, Wang et al. *Biomechanics and Modeling in Mechanobiology*

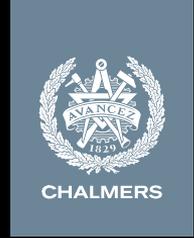


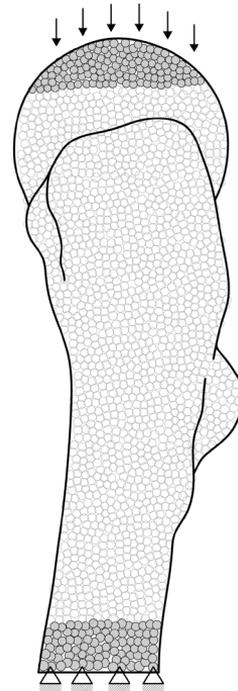
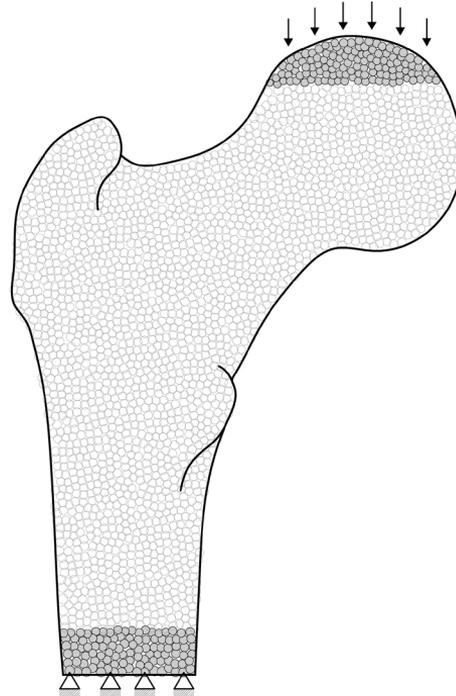
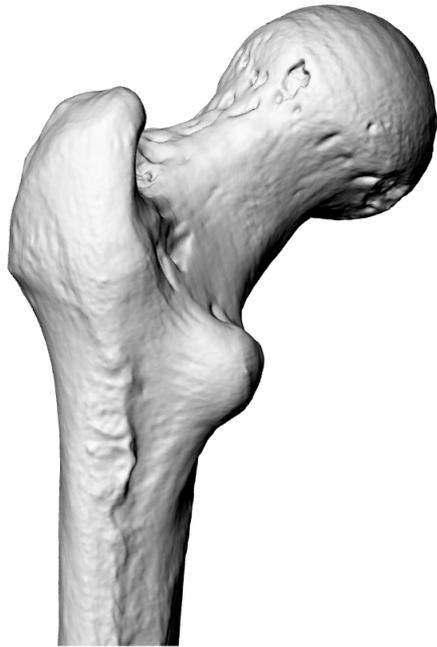


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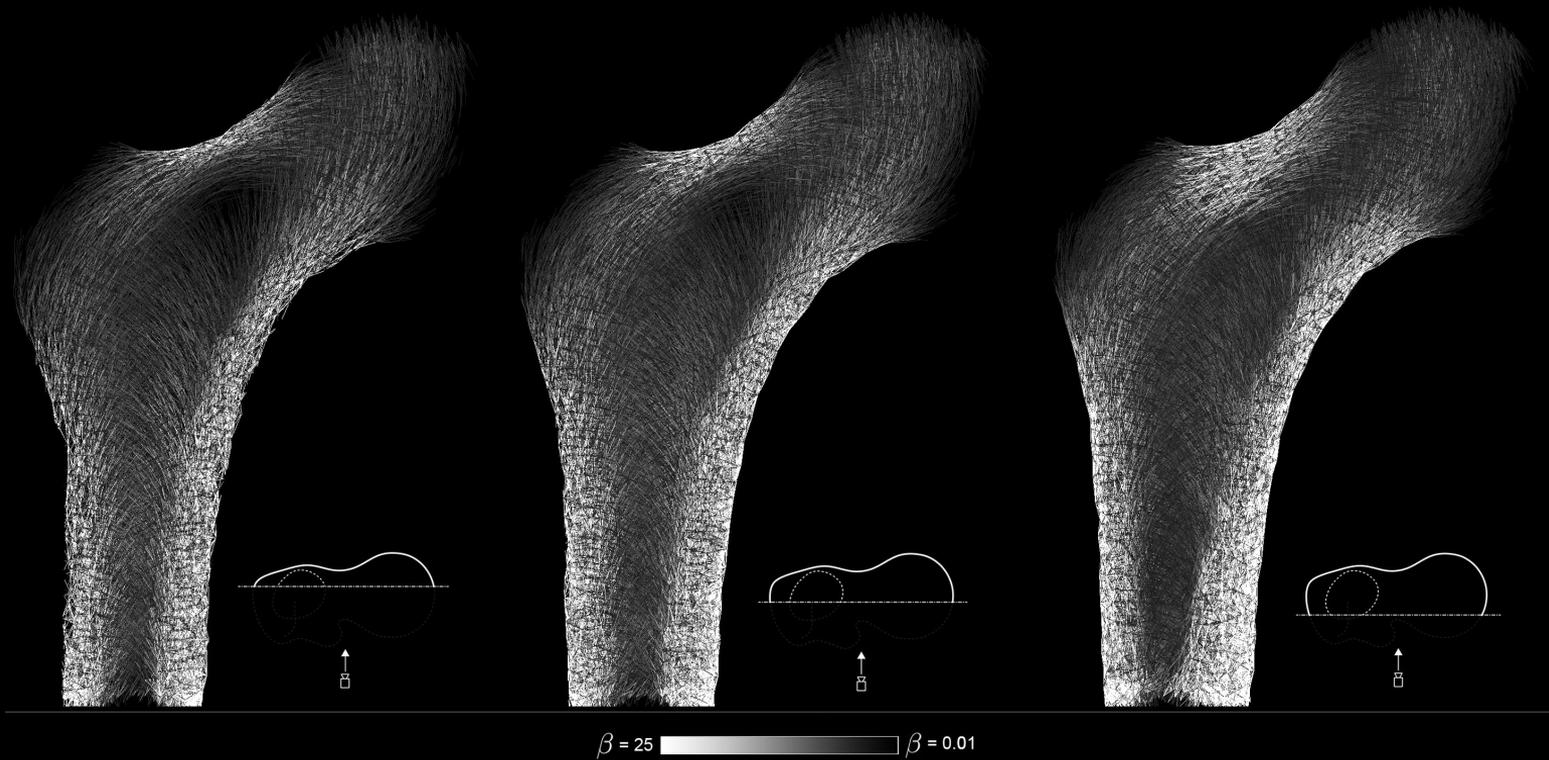






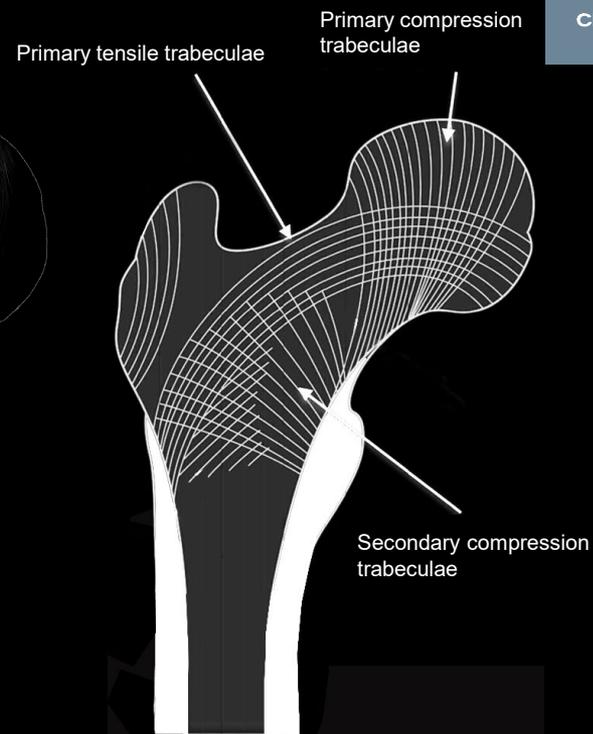
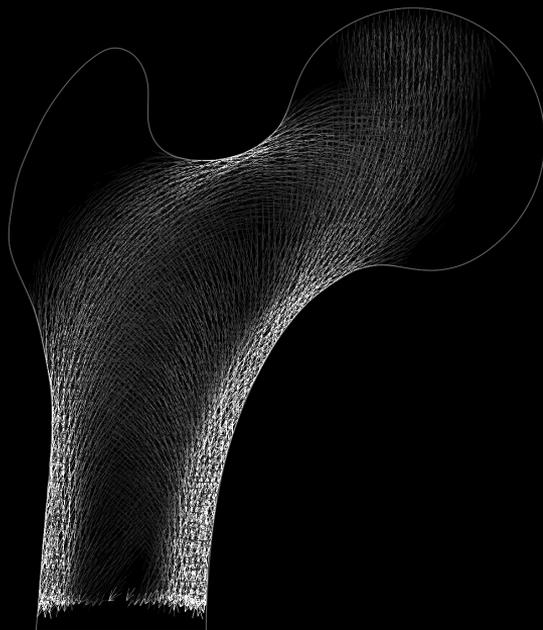




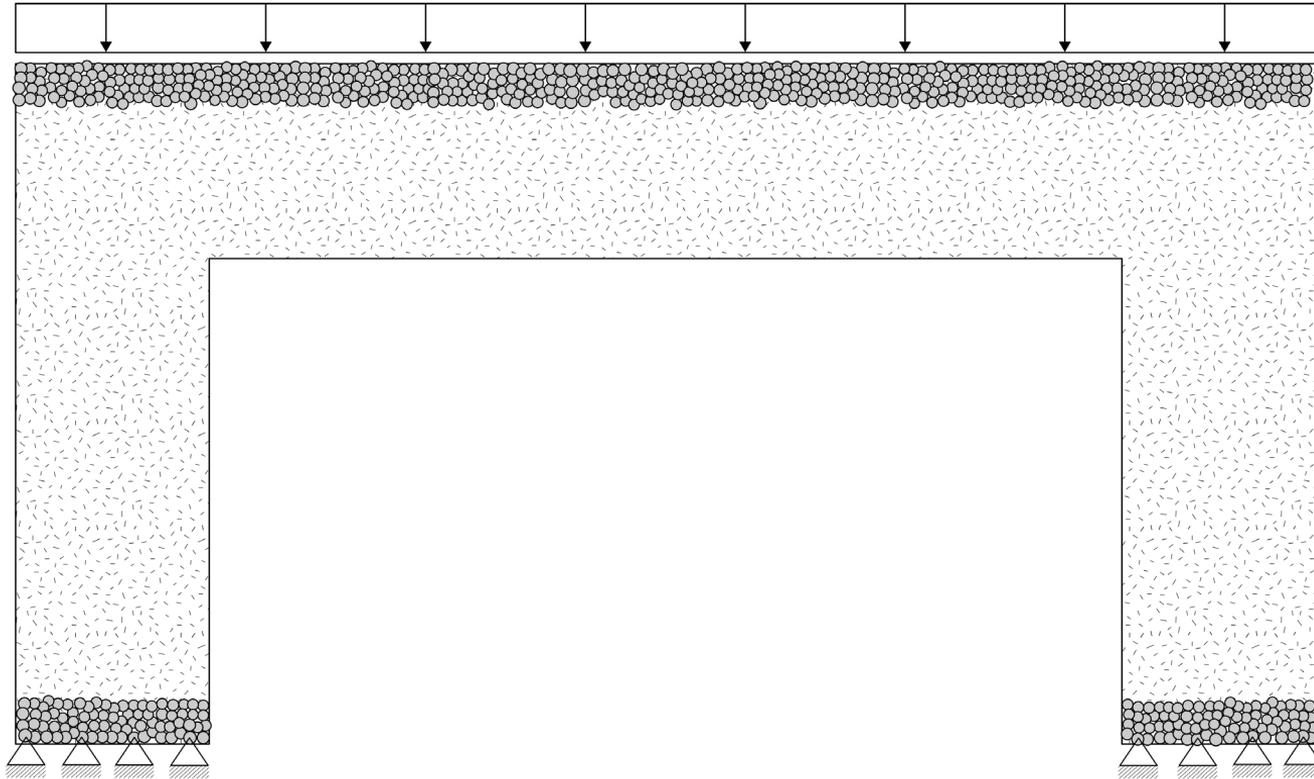




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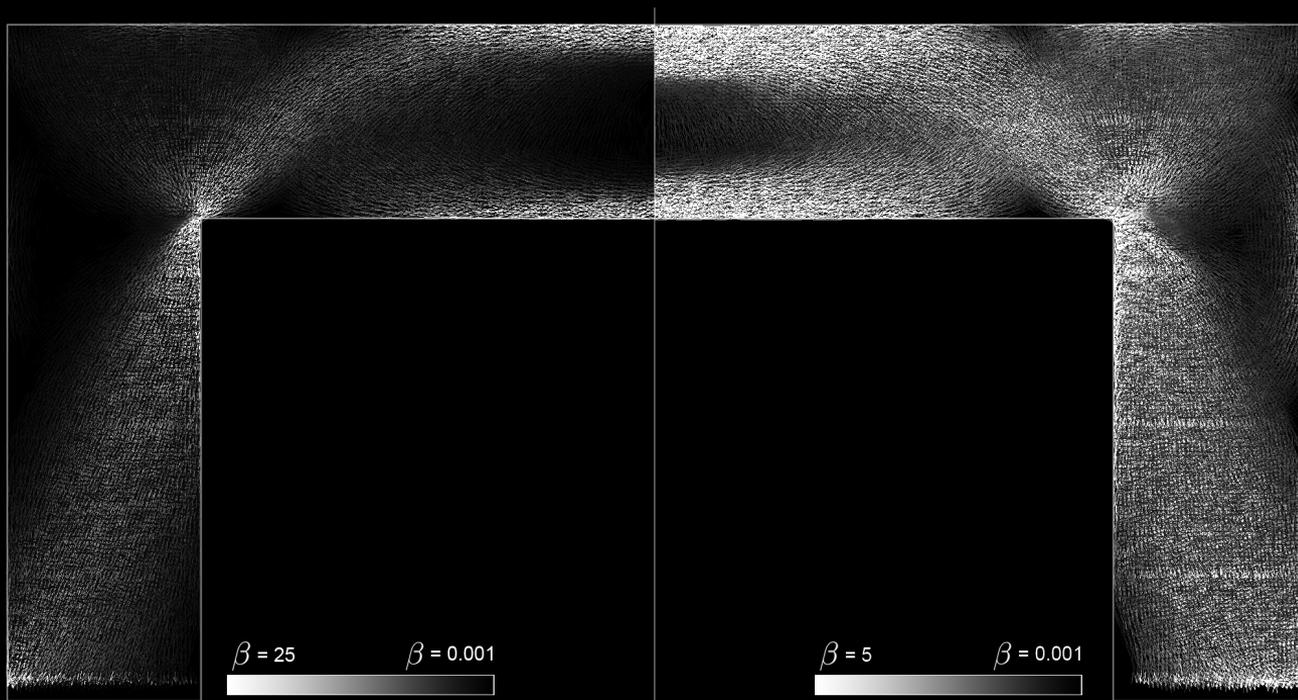


Reference: Hao, Wei et al. *Reverse wedge effect following intramedullary nail fixation of trochanteric fracture, what does it imply?* BMC Musculoskeletal Disorders





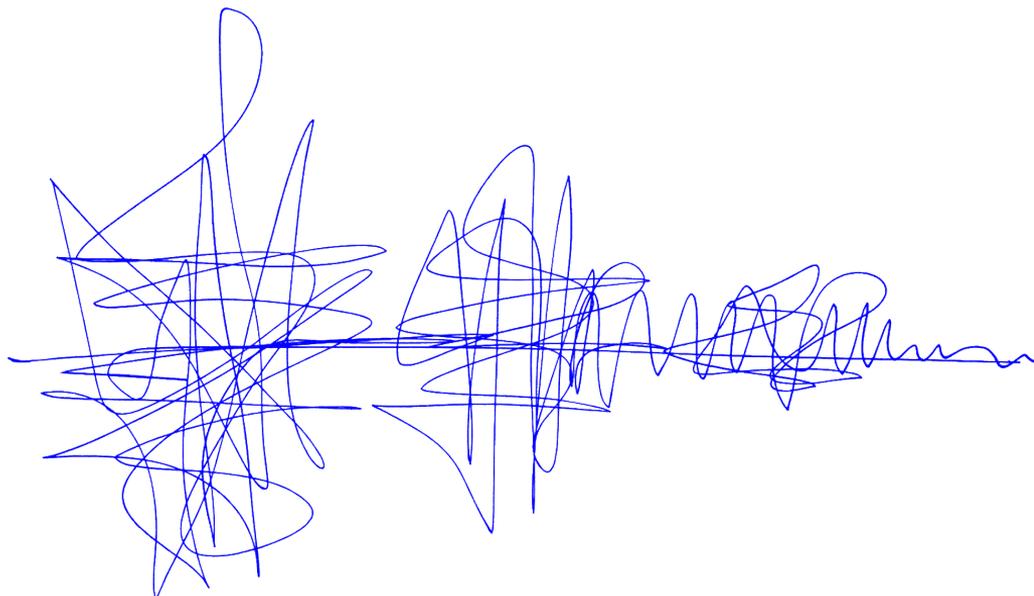
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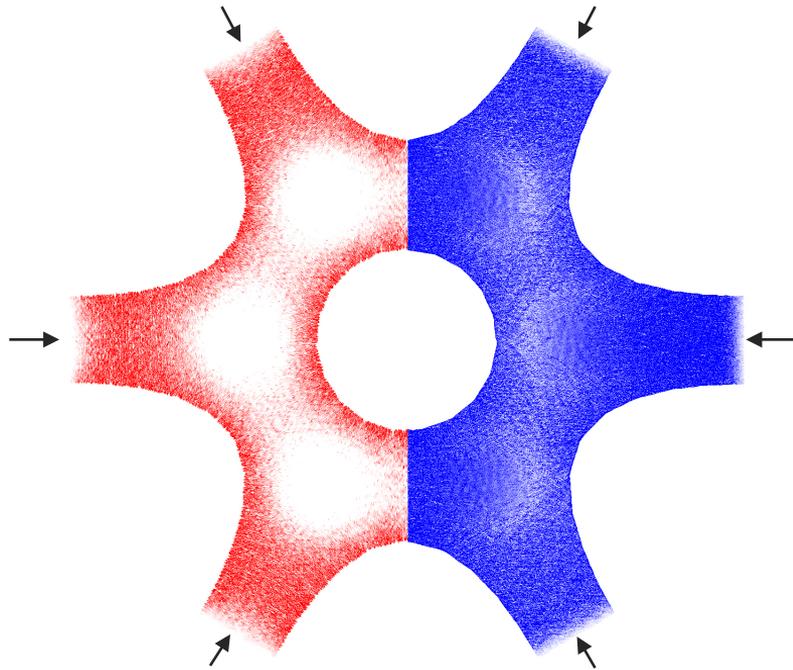


Particles: 24 840  
Arms: 676 218

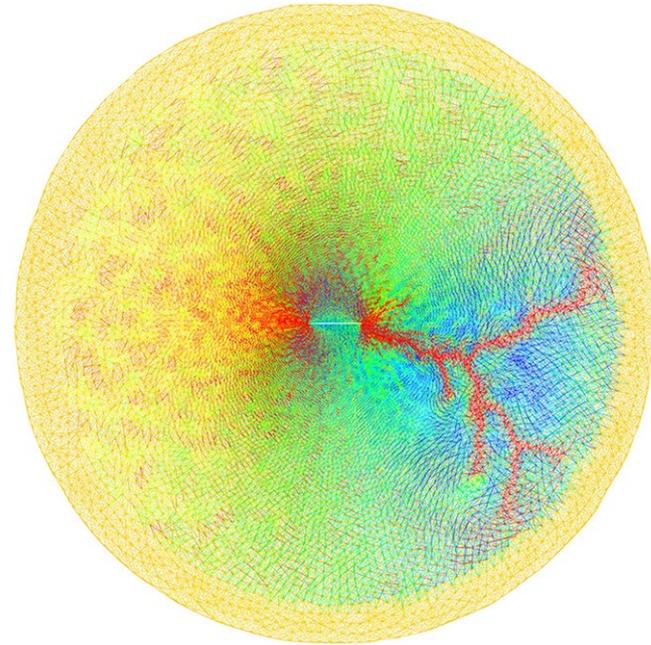


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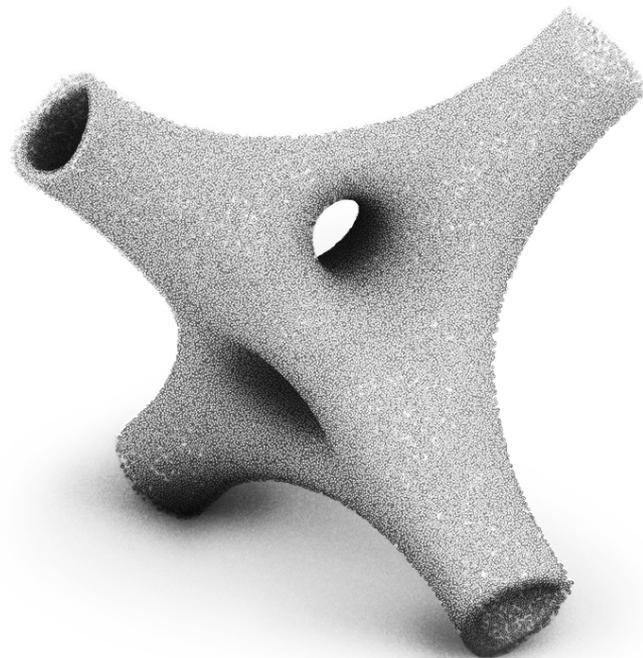
Tension in red compression in blue



Yielding and fracture



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